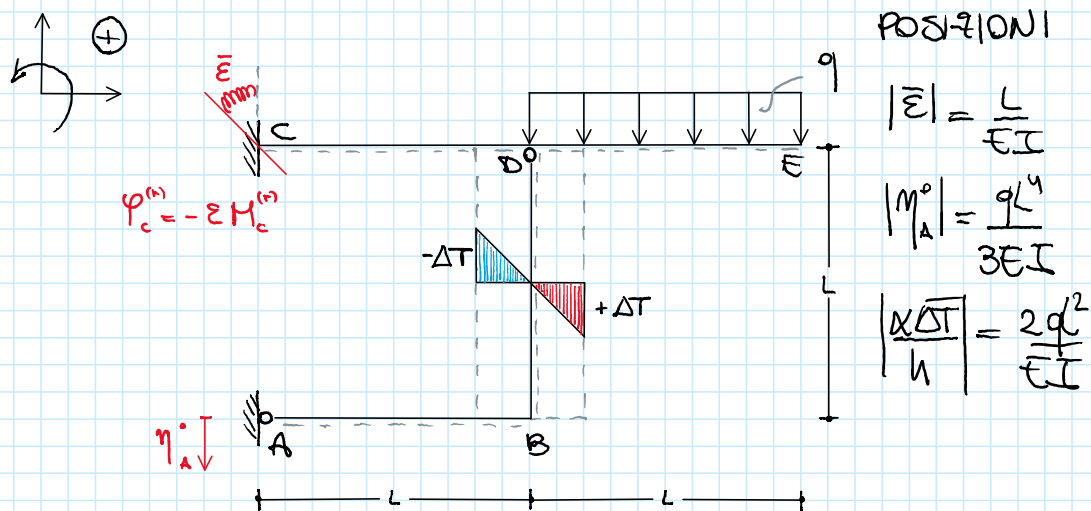
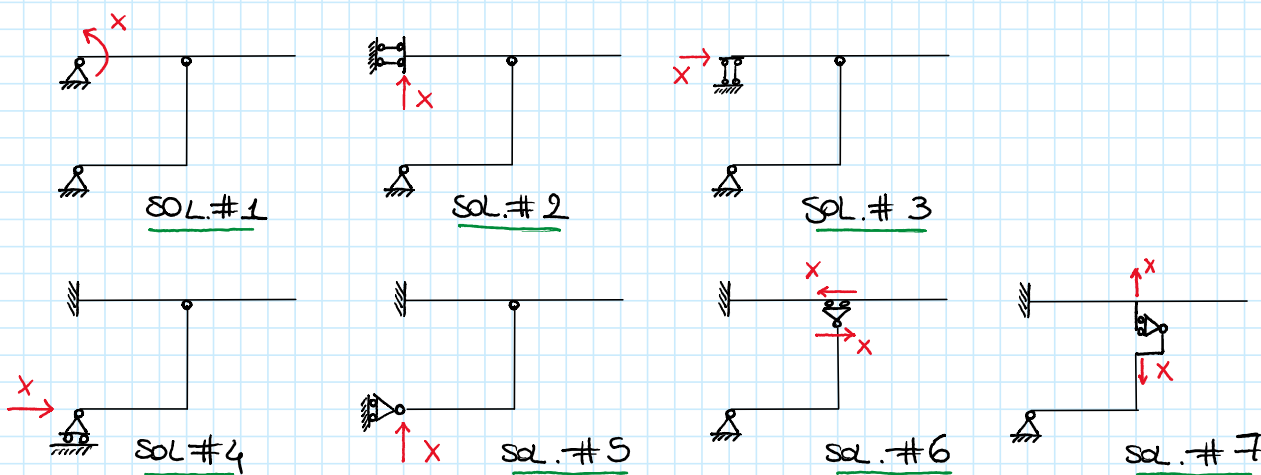


IPER 7

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA
DETERMINANDO IL DIAGRAMMA DEL MOMENTO

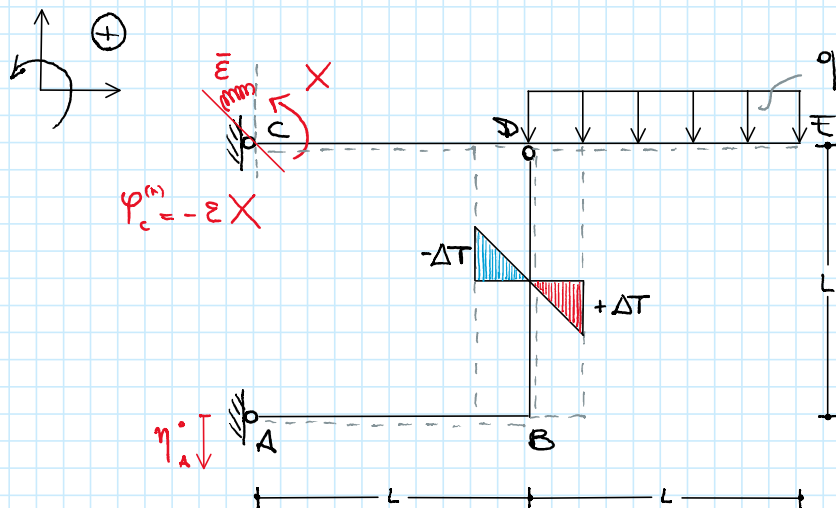


SOLUZIONI POSSIBILI:

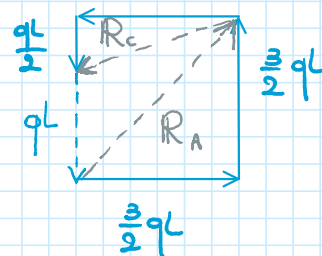
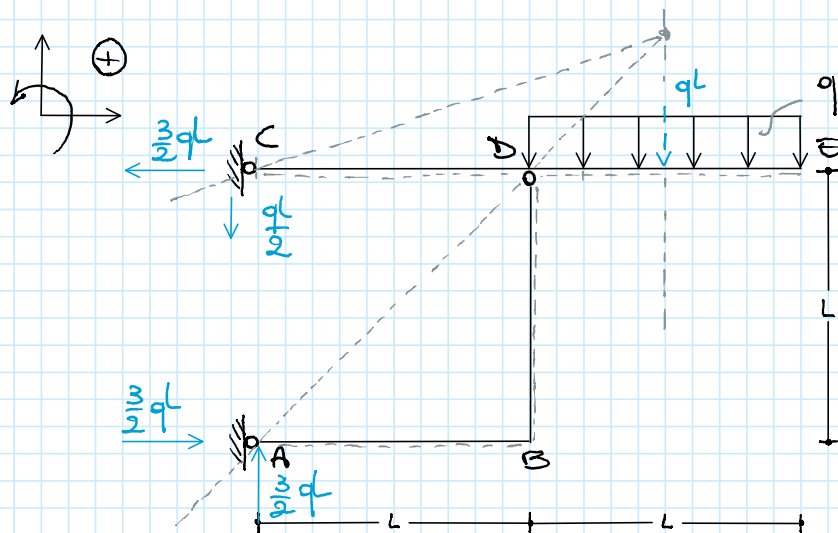


SOLUZIONE #1

SISTEMA PRINCIPALE IPERSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO AB $0 \leq z \leq L$

$\frac{3}{2}qL \rightarrow$ A $\leftarrow \frac{3}{2}qL$ \leftarrow $M^{(0)}(z) = \frac{3}{2}qLz$ $\left\{ \begin{array}{l} M_A = 0 \\ M_B = \frac{3}{2}qL^2 \end{array} \right.$

TRATTO BD $0 \leq z \leq L$

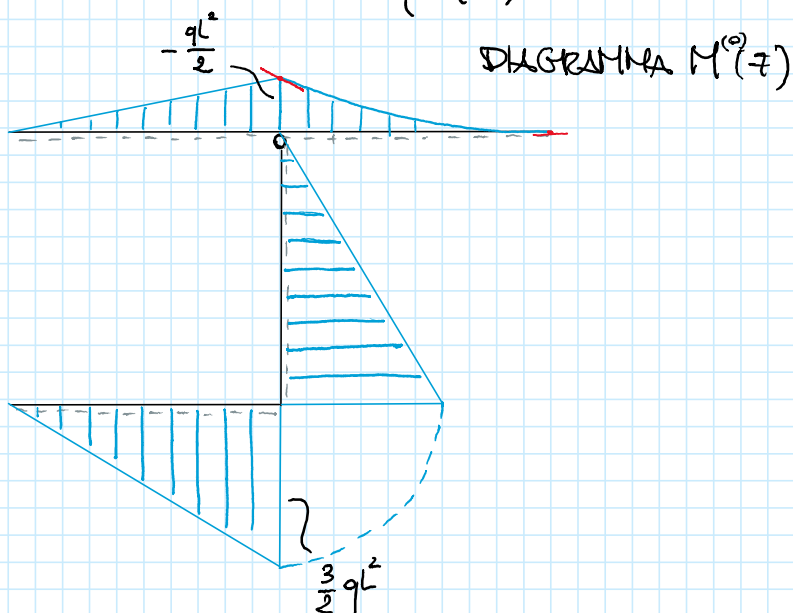
$\frac{3}{2}qL \rightarrow$ B $\leftarrow \frac{3}{2}qL$ \leftarrow $M^{(0)}(z) = \frac{3}{2}qL^2 - \frac{3}{2}qLz$ $\left\{ \begin{array}{l} M_B = \frac{3}{2}qL^2 \\ M_D = 0 \end{array} \right.$

TRATTO CD $0 \leq z \leq L$

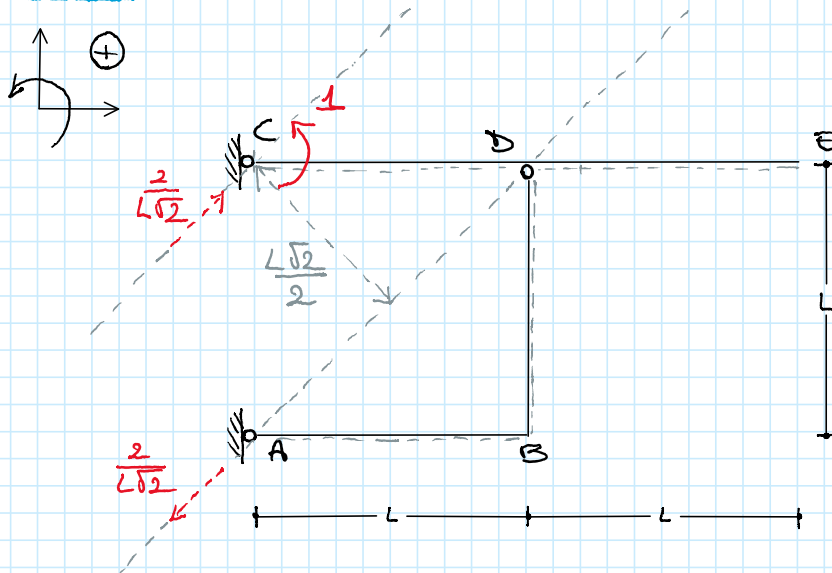
$\frac{3}{2}qL \leftarrow$ C $\rightarrow \frac{3}{2}qL$ \leftarrow $M^{(0)}(z) = -\frac{qL}{2}z$ $\left\{ \begin{array}{l} M_C = 0 \\ M_D = -\frac{qL^2}{2} \end{array} \right.$

TRATTO DE $L \leq z \leq 2L$

\rightarrow $\frac{q}{2}(2L-z)$ \leftarrow $M^{(0)}(z) = -\frac{q}{2}(2L-z)^2$ $\left\{ \begin{array}{l} M_D = -\frac{qL^2}{2} \\ M_E = 0 \end{array} \right.$



SCHEMA [1] SOLO $X=1$



TRATTO AB $0 \leq z \leq L$

$$\Rightarrow \begin{array}{c} \text{A} \\ \text{---} z \text{---} \\ \text{B} \end{array} \quad \begin{array}{c} \text{---} \frac{1}{L} \text{---} \\ \text{---} \frac{2}{L\sqrt{2}} \text{---} \end{array} \quad \Rightarrow \quad \boxed{M''(z) = -\frac{z}{L}} \quad \begin{cases} M_A = 0 \\ M_B = -1 \end{cases}$$

TRATTO BD $0 \leq z \leq L$

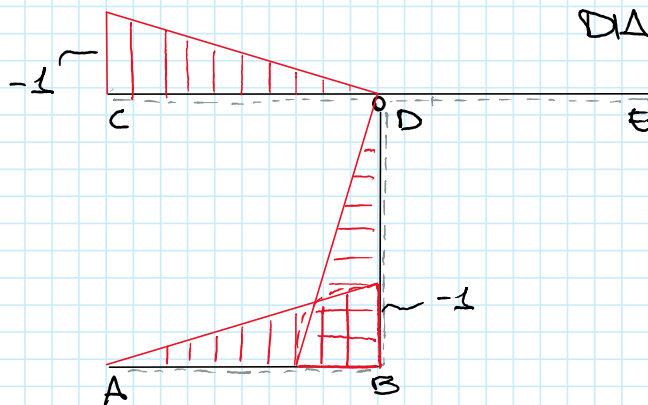
$$\Rightarrow \begin{array}{c} \text{D} \\ \text{---} \frac{1}{L} \text{---} \\ \text{---} L-z \text{---} \\ \text{---} \frac{1}{L} \text{---} \end{array} \quad \Rightarrow \quad \boxed{M''(z) = -\frac{1}{L}(L-z)} \quad \begin{cases} M_B = -1 \\ M_D = 0 \end{cases}$$

TRATTO CD $0 \leq z \leq L$

$$\begin{array}{c} \text{D} \\ \text{---} \frac{1}{L} \text{---} \\ \text{---} \frac{2}{L\sqrt{2}} \text{---} \\ \text{---} \frac{1}{L} \text{---} \end{array} \quad \begin{array}{c} \text{---} z \text{---} \\ \text{---} \frac{1}{L} \text{---} \end{array} \quad \Rightarrow \quad \boxed{M''(z) = -1 + \frac{z}{L}} \quad \begin{cases} M_C = -1 \\ M_D = 0 \end{cases}$$

TRATTO DE Seuico

DIAGRAMMA $M''(z)$



$$\begin{aligned}\underline{\Delta v_e} &= X_i \cdot \eta_i^{(r)} + \sum_j R_j^{(r)} \cdot \eta_j^{(r)} = \\ &= 1 \cdot \eta_c^{(r)} + R_{y_A} \cdot \eta_A^0 = -\bar{\varepsilon} X + \frac{M_A^0}{L}\end{aligned}$$

$-\bar{\varepsilon} X \leftarrow$

$$\begin{aligned}\underline{\Delta v_i} &= \int_{Str} M^{(r)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(r)} \frac{\alpha \Delta T}{h} dStr = \\ &= \int_{Str} M^{(r)} \frac{M^{(r)}}{EI} dStr + X \int_{Str} \frac{[M^{(r)}]^2}{EI} dStr + \int_{Str} M^{(r)} \frac{\alpha \Delta T}{h} dStr = \\ &= \frac{1}{EI} \left\{ \int_{AB} \left(-\frac{z}{L} \right) \left(\frac{3}{2} q L z \right) dz + \int_{BD} \left(-\frac{1}{L} (L-z) \right) \left[\frac{3}{2} q L^2 - \frac{3}{2} q L z \right] dz + \int_{CD} \left(-1 + \frac{z}{L} \right) \left(-\frac{q L}{2} z \right) dz \right\} + \\ &+ \frac{X}{EI} \left\{ \int_{AB} \left(-\frac{z}{L} \right)^2 dz + \int_{BD} \left(-\frac{1}{L} (L-z) \right)^2 dz + \int_{CD} \left(\frac{z}{L} - 1 \right)^2 dz \right\} + \int_{BD} \left(\frac{z}{L} - 1 \right) \left(\frac{\alpha \Delta T}{h} \right) dz = \\ &= \frac{1}{EI} \left\{ -\frac{3}{2} q \left[\frac{z^3}{3} \right]_0^L + \frac{3}{2} q L \left[\frac{z^2}{2} \right]_0^L - \frac{3}{2} q \left[\frac{z^3}{3} \right]_0^L - \frac{3}{2} q L^2 \left[z \right]_0^L + \frac{3}{2} q L \left[\frac{z^2}{2} \right]_0^L + \right. \\ &+ \left. \frac{q L}{2} \left[\frac{z^2}{2} \right] - \frac{q}{2} \left[\frac{z^3}{3} \right] \right\} + \frac{X}{EI} \left\{ \frac{1}{L^2} \left[\frac{z^3}{3} \right]_0^L + \left[z \right]_0^L + \frac{1}{L^2} \left[\frac{z^3}{3} \right]_0^L - \frac{2}{L} \left[\frac{z^2}{2} \right]_0^L + \frac{1}{L^2} \left[\frac{z^3}{3} \right]_0^L + \right. \\ &+ \left. \left[z \right]_0^L - \frac{2}{L} \left[\frac{z^2}{2} \right]_0^L \right\} + \frac{\alpha \Delta T}{h} \left\{ \frac{1}{L} \left[\frac{z^2}{2} \right]_0^L - \left[z \right]_0^L \right\} = \\ &= \frac{1}{EI} \left\{ -\frac{3}{2} q \frac{L^3}{3} + \frac{3}{2} q L \frac{L^2}{2} - \frac{3}{2} q \frac{L^3}{3} - \frac{3}{2} q L^2 L + \frac{3}{2} q L \frac{L^2}{2} + \frac{q L}{2} \frac{L^2}{2} - \frac{q}{2} \frac{L^3}{3} \right\} + \\ &+ \frac{X}{EI} \left\{ \frac{1}{L^2} \frac{L^3}{3} + 2 \left(\frac{1}{L^2} \frac{L^3}{3} + L - \frac{2}{L} \frac{L^2}{2} \right) \right\} + \frac{\alpha \Delta T}{h} \left\{ \frac{1}{L} \cdot \frac{L^2}{2} - L \right\} = \\ &= + \frac{q L^3}{EI} \left\{ -\frac{1}{2} + \frac{3}{4} - \frac{1}{2} - \frac{3}{2} + \frac{3}{4} + \frac{1}{4} - \frac{1}{6} \right\} + \frac{X}{EI} \left\{ + \frac{L}{3} + \frac{2}{3} L \right\} - \frac{\alpha \Delta T}{h} \frac{L}{2} = \\ &= -\frac{11}{12} \frac{q L^3}{EI} + \frac{X L}{EI} - \frac{\alpha \Delta T}{h} \frac{L}{2}\end{aligned}$$

$\Delta_{ve} = \Delta_{vi}$ fornisce

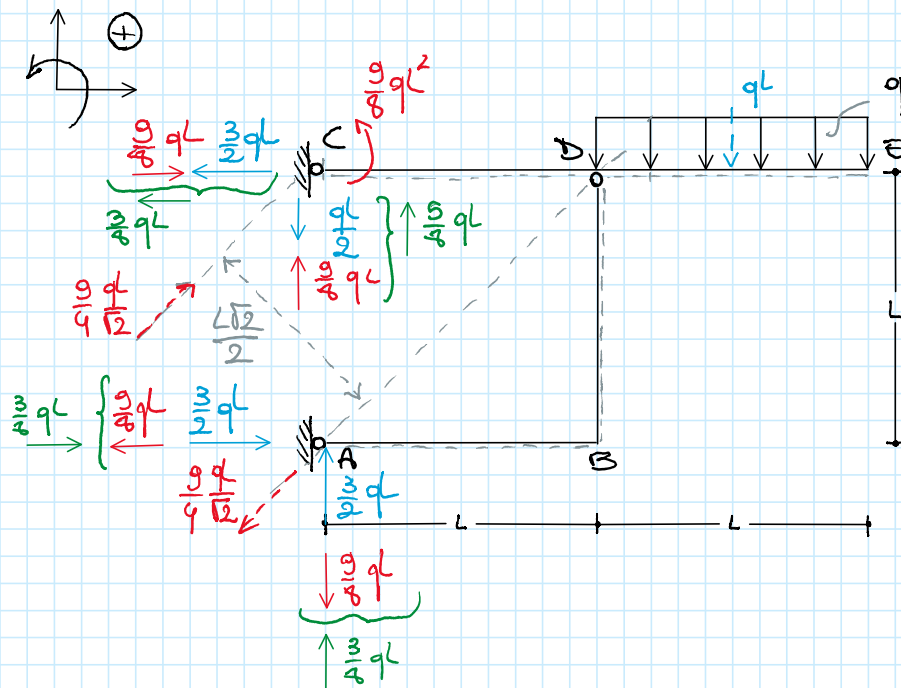
$$-\Sigma X + \frac{qL^3}{L} = -\frac{11}{12} \frac{qL^3}{EI} + \frac{XL}{EI} - \frac{\alpha \Delta T}{h} \frac{L}{2}$$

$$-\frac{L}{EI} X + \frac{qL^3}{3EI} = -\frac{11}{12} \frac{qL^3}{EI} + X \frac{L}{EI} - \frac{qL^3}{EI}$$

$$qL^3 \left\{ \frac{1}{3} + \frac{11}{12} + 1 \right\} = 2X \quad X = \frac{9}{8} qL^2 \text{ POSITIVO}$$

Verso ipotizzato ok!

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



TRATTO AB $0 \leq z \leq L$

$\frac{3}{8}qL$ \rightarrow A \leftarrow $\frac{3}{8}qL$ \rightarrow \Rightarrow $M^{(r)}(z) = \frac{3}{8}qLz$ $\begin{cases} M_A = 0 \\ M_B = \frac{3}{8}qL^2 \end{cases}$

TRATTO BD $0 \leq z \leq L$

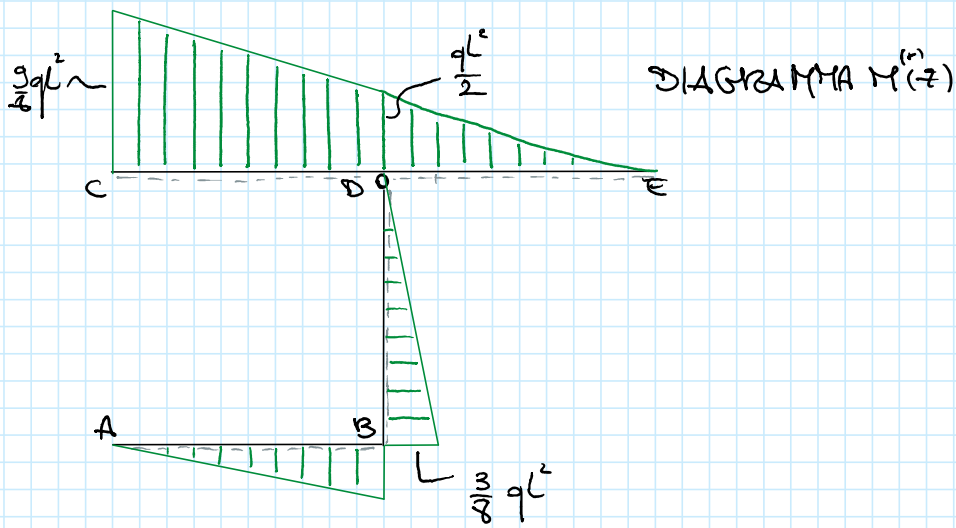
\Rightarrow \leftarrow $\frac{3}{8}qL$ \leftarrow $\frac{3}{8}qL$ \rightarrow \Rightarrow $M^{(r)}(z) = \frac{3}{8}qL(L-z)$ $\begin{cases} M_B = \frac{3}{8}qL^2 \\ M_D = 0 \end{cases}$

TRATTO CD $0 \leq z \leq L$

$\frac{3}{8}qL$ \leftarrow C \leftarrow $\frac{9}{8}qL^2$ \leftarrow $\frac{5}{8}qL$ \rightarrow \Rightarrow $M^{(r)}(z) = \frac{5}{8}qLz - \frac{9}{8}qL^2$ $\begin{cases} M_C = -\frac{9}{8}qL^2 \\ M_D = -\frac{qL^2}{2} \end{cases}$

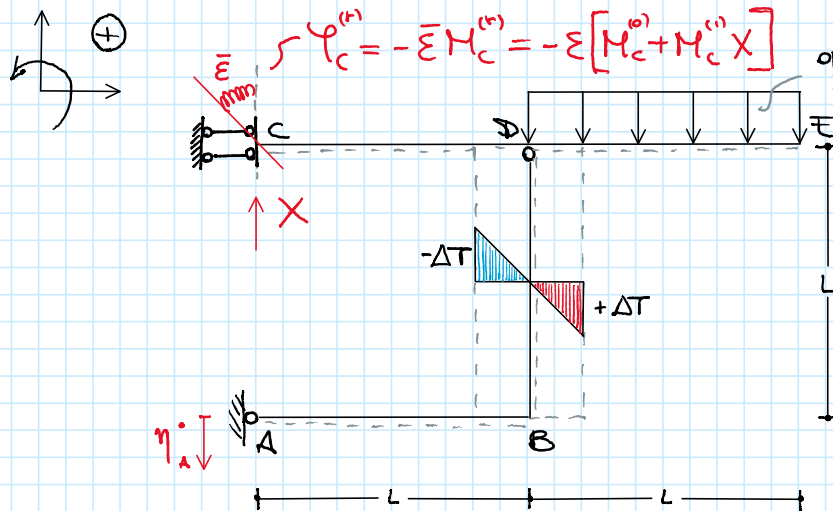
TRATTO DE $L \leq z \leq 2L$

\Rightarrow \leftarrow $\frac{q}{2}(2L-z)^2$ \leftarrow $\frac{q}{2}(2L-z)^2$ \rightarrow \Rightarrow $M^{(r)}(z) = -\frac{q}{2}(2L-z)^2$ $\begin{cases} M_D = -\frac{qL^2}{2} \\ M_E = 0 \end{cases}$

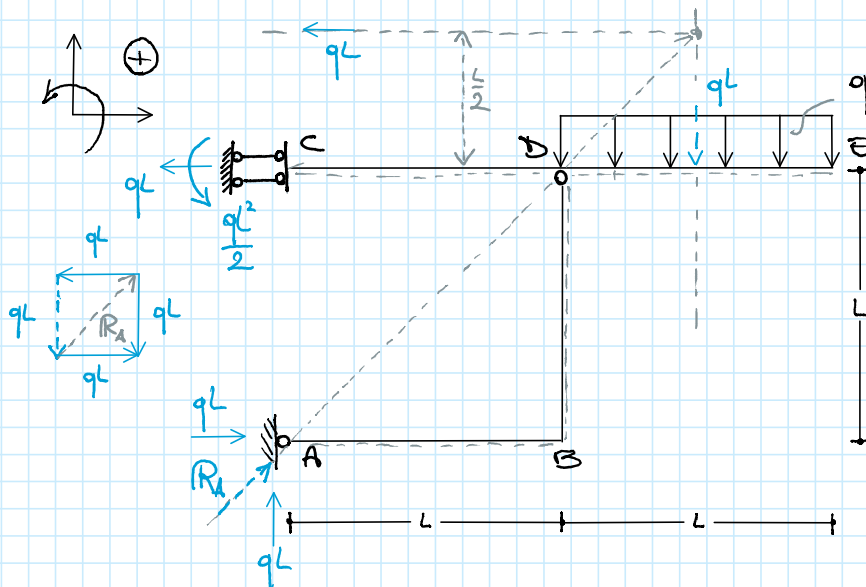


SOLUZIONE #2

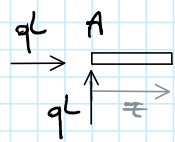
SISTEMA PRINCIPALE IPOTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



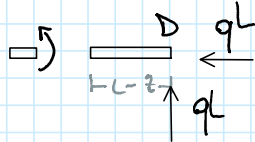
TRATTO AB $0 \leq z \leq L$



$$M^{(0)}(z) = qLz$$

$$\begin{cases} M_A = 0 \\ M_B = qL^2 \end{cases}$$

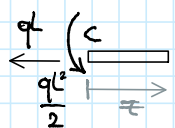
TRATTO BD $0 \leq z \leq L$



$$M^{(0)}(z) = qL(L-z)$$

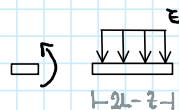
$$\begin{cases} M_D = qL^2 \\ M_B = 0 \end{cases}$$

TRATTO CD $0 \leq z \leq L$



$$M^{(0)}(z) = -\frac{qL^2}{2}$$

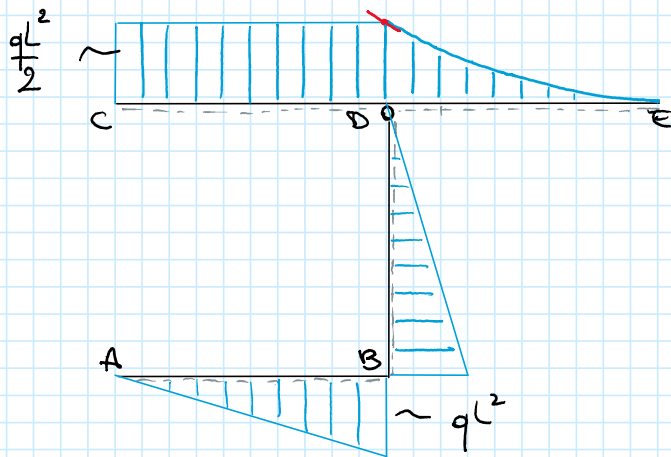
TRATTO DE $L \leq z \leq 2L$



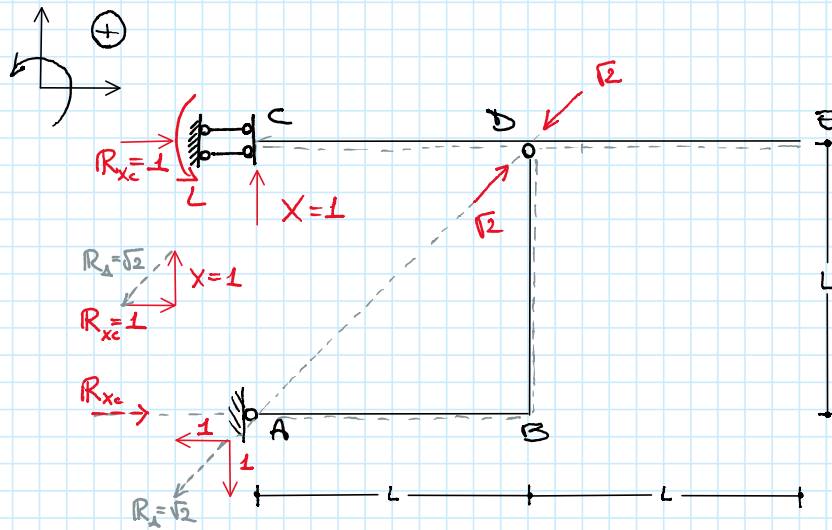
$$M^{(0)}(z) = -\frac{q}{2}(2L-z)^2$$

$$\begin{cases} M_D = -\frac{qL^2}{2} \\ M_E = 0 \end{cases}$$

DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $X=1$



TRATTO AB $0 \leq z \leq L$

$$\begin{aligned} & \text{Diagram of segment AB with length } L \text{ and coordinate } z. \\ & \Rightarrow M''(z) = -z \end{aligned} \quad \begin{cases} M_A = 0 \\ M_B = -L \end{cases}$$

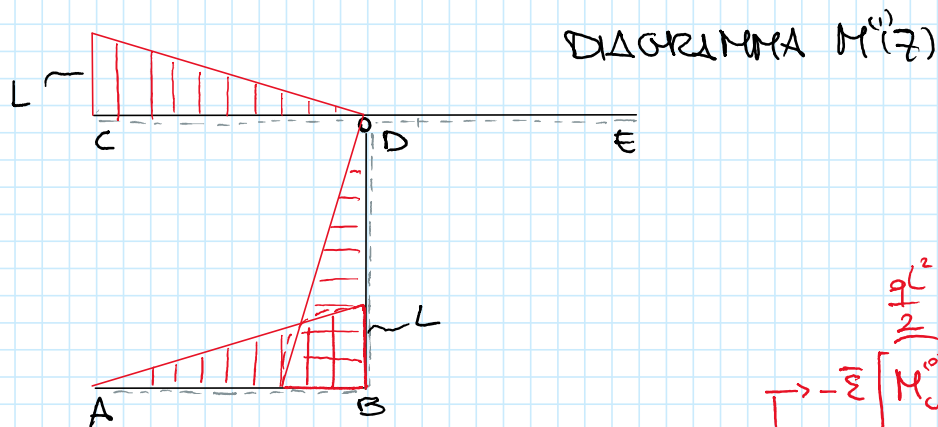
TRATTO BD $0 \leq z \leq L$

$$\Rightarrow \text{Diagram of segment BD with length } L \text{ and coordinate } z. \quad M''(z) = -(L-z) \quad \begin{cases} M_B = -L \\ M_D = 0 \end{cases}$$

TRATTO CD $0 \leq z \leq L$

$$\Rightarrow \text{Diagram of segment CD with length } L \text{ and coordinate } z. \quad M''(z) = -L+z \quad \begin{cases} M_C = -L \\ M_D = 0 \end{cases}$$

TRATTO DE Seaisco



$$\begin{aligned} L_{ve} &= 1 \cdot \eta_i + \sum_j R_j^{(n)} \eta_j^{(n)} = \underbrace{R_{yA}^{(1)}}_{-1} \cdot \underbrace{[-\eta_A^0]}_{L} + \underbrace{M_C^{(1)}}_L \cdot \underbrace{\varphi_C^{(1)}}_{\frac{9L^2}{2}} = \\ &= \underbrace{\eta_A^0}_{-1} - \bar{E} L \left[\frac{9L^2}{2} + XL \right] \end{aligned}$$

$$\rightarrow -\bar{E} \left[M_C^{(0)} + X M_C^{(1)} \right]$$

$$\begin{aligned}
\underline{L_{VI}} &= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr = \\
&= \frac{1}{EI} \int_{Str} M^{(1)} M^{(0)} dStr + \frac{\alpha}{EI} \int_{Str} [M^{(1)}]^2 dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr = \\
&= \frac{1}{EI} \left\{ \int_{AB} M^{(1)} M^{(0)} dz + \int_{BD} M^{(1)} M^{(0)} dz + \int_{CD} M^{(1)} M^{(0)} dz \right\} + \\
&+ \frac{\alpha}{EI} \left\{ \int_{AB} [M^{(1)}]^2 dz + \int_{BD} [M^{(1)}]^2 dz + \int_{CD} [M^{(1)}]^2 dz \right\} + \int_{BD} M^{(1)} \frac{\alpha \Delta T}{h} dz = \\
&= \frac{1}{EI} \left\{ \int_0^L [-z][qLz] dz + \int_0^L qL[L-z][z-L] dz + \right. \\
&+ \left. \int_0^L -\frac{qL^2}{2}[z-L] dz \right\} + \frac{\alpha}{EI} \left\{ \int_0^L z^2 dz + \int_0^L (z-L)^2 dz + \int_0^L (z-L)^2 dz \right\} + \\
&+ \frac{\alpha \Delta T}{h} \int_0^L (z-L) dz = \\
&= \frac{1}{EI} \left\{ \int_0^L -qLz^2 dz + \int_0^L qL[-z^2 - L^2 + 2zL] dz + \int_0^L -\frac{qL^2}{2}[z-L] dz \right\} + \\
&+ \frac{\alpha}{EI} \left\{ \int_0^L z^2 dz + 2 \int_0^L (z^2 + L^2 - 2zL) dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L (z-L) dz = \\
&= \frac{1}{EI} \left\{ -qL \cdot \frac{L^3}{3} + qL \left[-\frac{L^3}{3} - L^3 + 2 \frac{L^3}{2} \right] - \frac{qL^2}{2} \left[\frac{L^2}{2} - L^2 \right] \right\} + \\
&+ \frac{\alpha}{EI} \left\{ \frac{L^3}{3} + 2 \left[\frac{L^3}{3} + L^3 - 2 \frac{L^3}{2} \right] \right\} + \frac{\alpha \Delta T}{h} \left[\frac{L^2}{2} - L^2 \right] = \\
&= \frac{1}{EI} \left\{ -\frac{qL^4}{3} - \frac{qL^4}{3} - qL^4 + qL^4 + \frac{qL^4}{4} \right\} + \frac{\alpha}{EI} \left\{ \frac{L^3}{3} + \frac{2}{3} L^3 \right\} - \frac{L^2}{2} \frac{\alpha \Delta T}{h} = \\
&= \underline{\underline{-\frac{5}{12} \frac{qL^4}{EI} + \frac{\alpha}{EI} L^3 - \frac{\alpha \Delta T}{h} \frac{L^2}{2}}}
\end{aligned}$$

$\Delta v_e = \Delta v_i$ fornisce

$$\eta_A - \bar{\epsilon} L \left[\frac{qL^2}{2} + XL \right] = -\frac{5}{12} \frac{qL^4}{EI} + \frac{XL^3}{EI} - \frac{\alpha \Delta T}{h} \cdot \frac{L^2}{2}$$

$$\frac{qL^4}{3EI} - \frac{L^2}{EI} \cdot \frac{qL^2}{2} - \frac{L^3}{EI} X = -\frac{5}{12} \frac{qL^4}{EI} + \frac{L^3}{EI} X - \frac{qL^4}{EI}$$

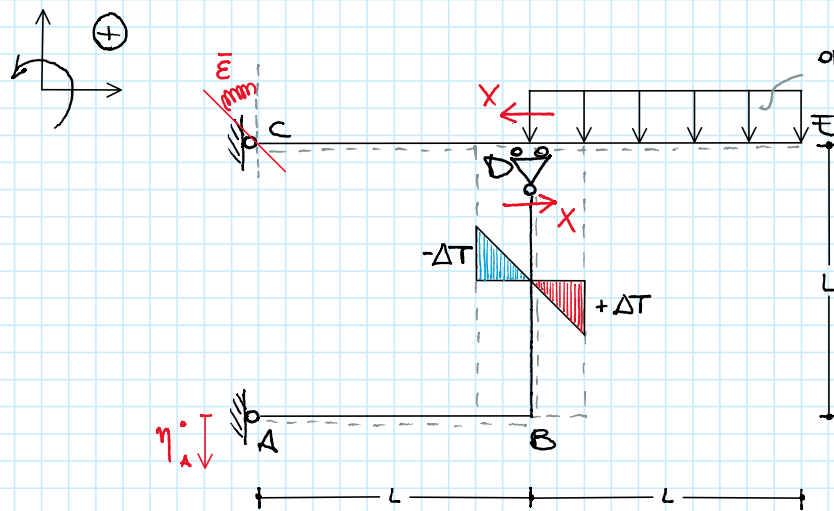
$$q \left[\frac{1}{3} - \frac{1}{2} + \frac{5}{12} + 1 \right] = X 2$$

$$\frac{15}{12} qL = X 2$$

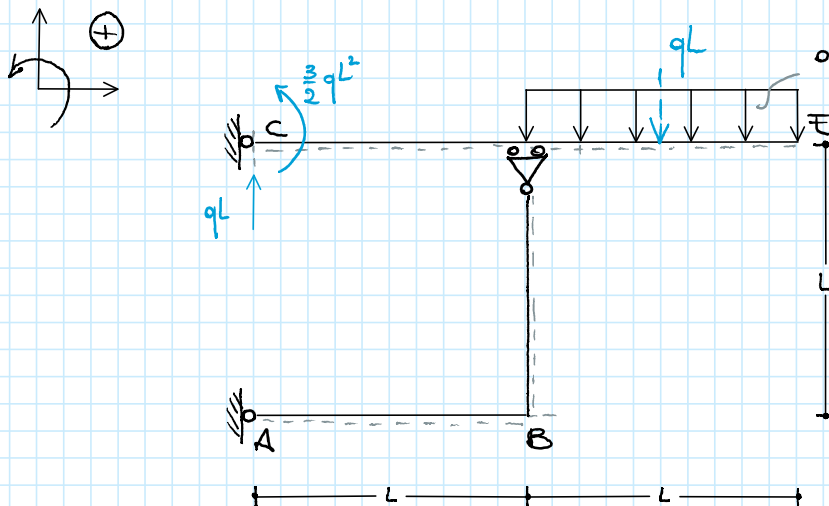
$$X = \frac{5}{8} qL \quad \text{POSITIVO, Verso corretto}$$

SOLUZIONE #6

• SISTEMA PRINCIPALE ISOSTATICO

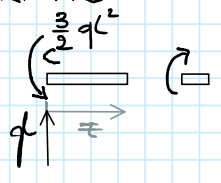


Schema [0] SOLO CARICHI ESTERNI



TRATTO AB } Scarichi: $M^{(0)}(z) = 0$
 TRATTO BC }

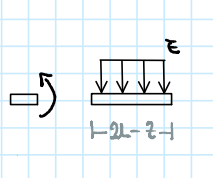
TRATTO CD $0 \leq z \leq L$



$$M^{(0)}(z) = -\frac{3}{2}qL^2 + qLz$$

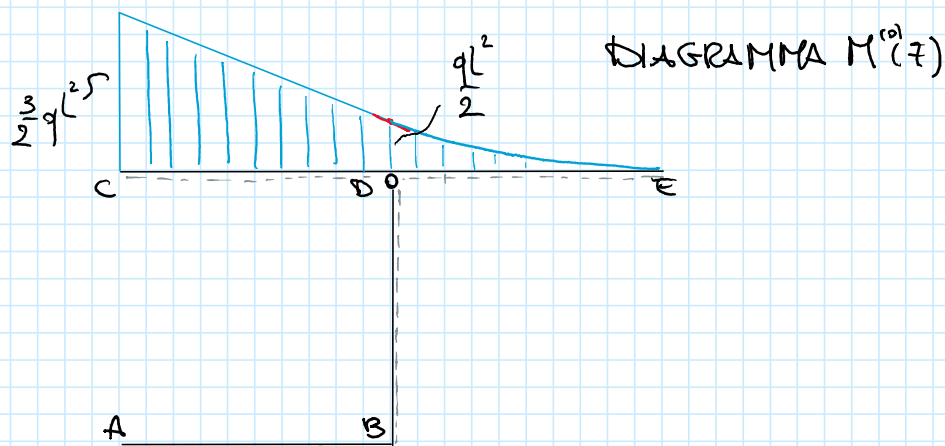
$$\begin{cases} M_C = -\frac{3}{2}qL^2 \\ M_D = -\frac{qL^2}{2} \end{cases}$$

TRATTO DE $L \leq z \leq 2L$

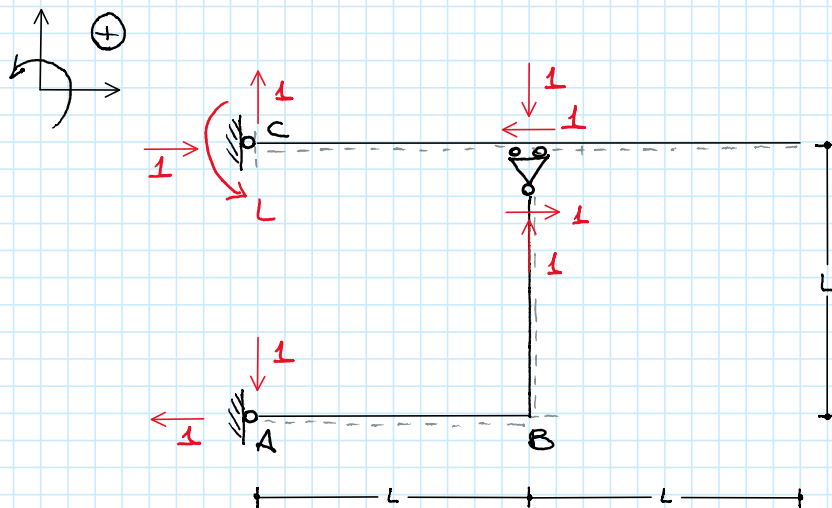


$$M^{(0)}(z) = -\frac{q}{2}(2L-z)^2$$

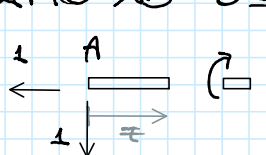
$$\begin{cases} M_D = -\frac{qL^2}{2} \\ M_E = 0 \end{cases}$$



SCHEMA [1] SOLO $X=1$




TRATTO AB $0 \leq z \leq L$



$$M^{(0)}(z) = -z$$

$$\begin{cases} M_A = 0 \\ M_B = -L \end{cases}$$

TRATTO BD $0 \leq z \leq L$

\Rightarrow 
 $M^{(n)}(z) = -(L-z)$
 $\begin{cases} M_B = -L \\ M_D = 0 \end{cases}$

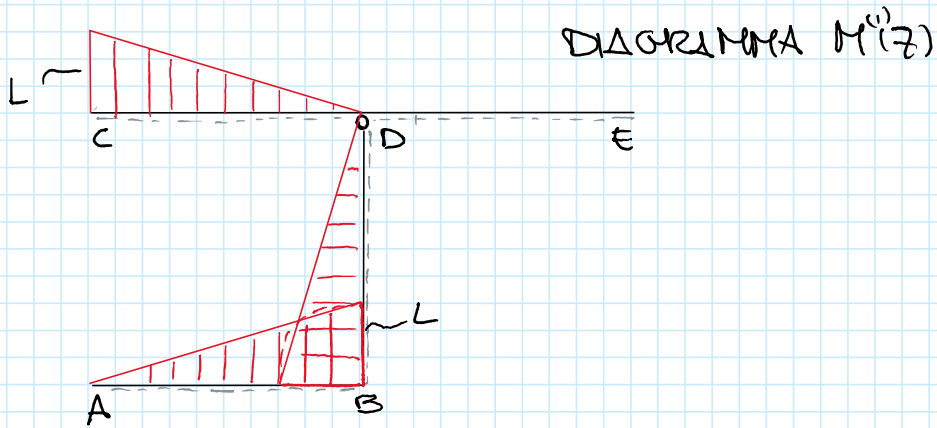
TRATTO CD $0 \leq z \leq L$

Diagram of a beam element of length L with forces F and F applied at the ends. The beam is shown in a deformed state with a curved arrow indicating rotation. The forces are labeled F and F .

$$M''(\xi) = -L + \xi$$

$$\begin{cases} M_c = -L \\ M_o = 0 \end{cases}$$

TIRATTO DE Seanico



$$\mathcal{L}_{ve} = 1 \cdot \eta_0 + \sum_j \left(R_j^{(n)} \eta_j^{(n)} \right) - \underbrace{\mathbb{E} \left[\frac{3}{2} q^2 + \chi L \right]}_{\substack{M_c^{(n)} \cdot \eta_c^{(n)} + R_{ga}^{(n)} \eta_a^{(n)} \\ + L \cdot \underbrace{-\mathbb{E} M_c^{(n)}}_{M_c^{(n)} + \chi M_c^{(n)}} - \eta_a^{(n)}}} \cdot \eta_0$$

$$\begin{aligned} \alpha_u &= \int_{s_{tr}} M'' \frac{M'''}{EI} ds_{tr} + \int_{s_{tr}} M''' \frac{\alpha \Delta T}{h} ds_{tr} = \\ &= \frac{1}{EI} \int_{s_{tr}} M'' M''' ds_{tr} + \frac{\alpha}{EI} \int_{s_{tr}} [M''']^2 ds_{tr} + \frac{\alpha \Delta T}{h} \int_{s_{tr}} M''' ds_{tr} \\ &= \frac{1}{EI} \left\{ \int_{AB} M'' M''' dz \right\} + \frac{\alpha}{EI} \left\{ \int_{AB} [M''']^2 dz + \int_{BD} [M''']^2 dz + \int_{CD} [M''']^2 dz \right\} + \frac{\alpha \Delta T}{h} \int_{BD} M''' dz = \\ &= \frac{1}{EI} \left\{ \int_0^L \underbrace{(z-L) \left(qLz - \frac{3}{2} qL^2 \right)}_{qLz^2 - \frac{3}{2} qL^2 z - qL^2 z + \frac{3}{2} qL^3} dz \right\} + \frac{\alpha}{EI} \left\{ \int_0^L z^2 dz + \int_0^L \underbrace{(L-z)^2}_{L^2 + z^2 - 2Lz} dz + \int_0^L (z-L)^2 dz \right\} + \\ &+ \frac{\alpha \Delta T}{h} \int_0^L (z-L) dz = \end{aligned}$$

$$= \frac{1}{EI} \left\{ qL \frac{L^3}{3} - \frac{3}{2} qL^2 \cdot \frac{L}{2} - qL^2 \frac{L}{2} + \frac{3}{2} qL^3 \cdot L \right\} + \frac{X}{EI} \left\{ \frac{L^3}{3} + 2 \left[L^3 + \frac{L^3}{3} - 2L \cdot \frac{L^2}{2} \right] \right\} + \frac{\alpha \Delta T}{h} \left\{ \frac{L^2}{2} - L^2 \right\} =$$

$$= \frac{qL^4}{EI} \left\{ \frac{1}{3} - \frac{3}{4} - \frac{1}{2} + \frac{3}{2} \right\} + \frac{X}{EI} L^3 - \frac{\alpha \Delta T}{h} \frac{L^2}{2} =$$

$$= \frac{7}{12} \frac{qL^4}{EI} + \frac{XL^3}{EI} - \frac{L^2}{2} \frac{\alpha \Delta T}{h}$$

$\Delta_{ve} = \Delta_{vi}$ fornisce

$$-\frac{3}{2} qL^3 \bar{E} - \bar{E} X L^2 + M_a^o = \frac{7}{12} \frac{qL^4}{EI} + \frac{L^3}{EI} X - \frac{L^2}{2} \frac{\alpha \Delta T}{h}$$

$$-\frac{3}{2} \frac{qL^4}{EI} - \frac{L^3}{EI} X + \frac{qL^4}{3EI} = \frac{7}{12} \frac{qL^4}{EI} + \frac{L^3}{EI} X - \frac{qL^4}{EI}$$

$$qL \left\{ -\frac{3}{2} + \frac{1}{3} - \frac{7}{12} + 1 \right\} = 2X$$

$$-\frac{9}{12} qL = 2X$$

$$X = -\frac{3}{4} qL \quad \text{NEGATIVO, verso opposto}$$