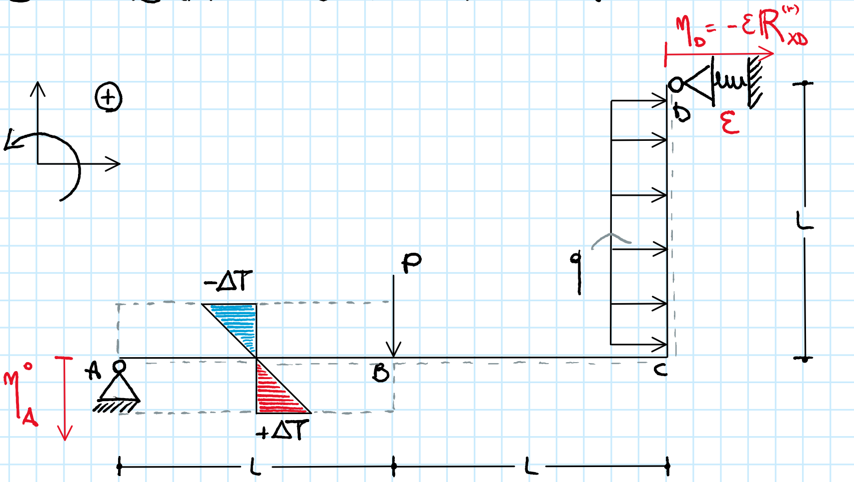
IPER 9

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA DETERMINANDO IL DIAGRAMMA DEL MOMENTO



POSIZIONI

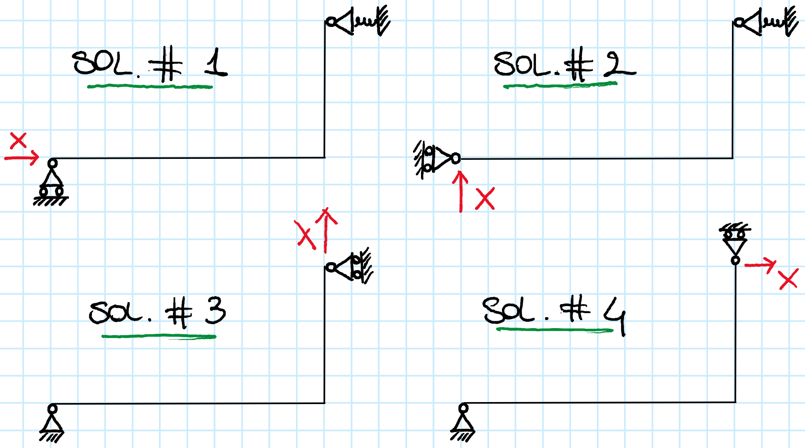
$|P| = qL$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{qL^2}{EI}$

$|\eta_A^0| = \frac{qL^4}{EI}$

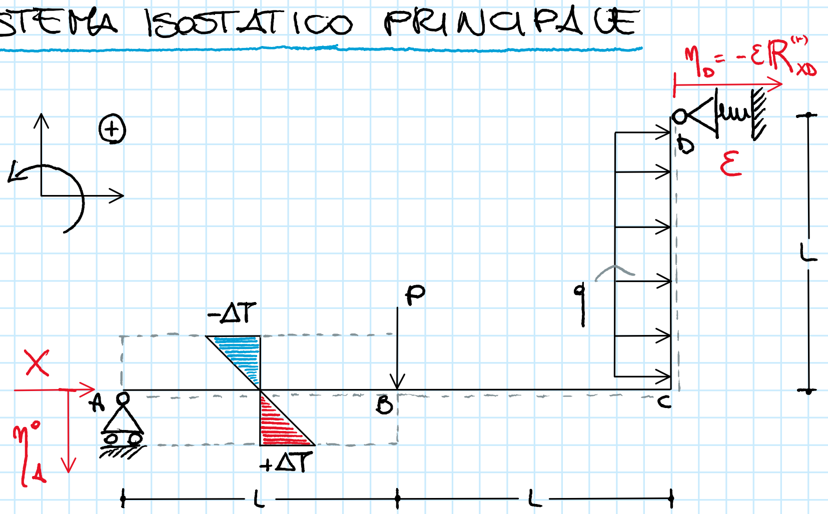
$|\varepsilon| = \frac{L^3}{EI}$

SOLUZIONI POSSIBILI

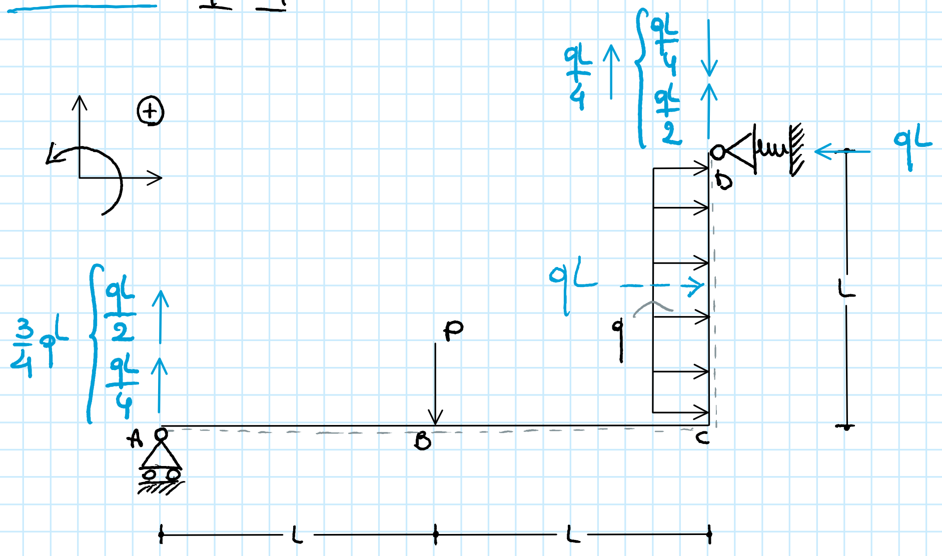


SOLUZIONE # 1

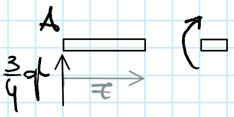
- SISTEMA ISOSTATICO PRINCIPALE



SCHEMA [0] SOLO CARICHI ESTERNI



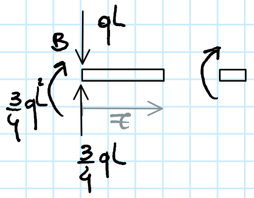
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{3}{4} qL z}$$

$$\begin{cases} M_A = 0 \\ M_B = \frac{3}{4} qL^2 \end{cases}$$

TRATTO BC $L \le z \le 2L$

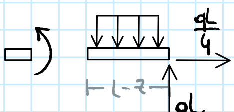


$$M^{(0)} = \frac{3}{4} qL^2 + \frac{3}{4} qL z - qL z =$$

$$\boxed{= -\frac{qL}{4} z + qL^2}$$

$$\begin{cases} M_B = \frac{3}{4} qL^2 \\ M_C = \frac{qL^2}{2} \end{cases}$$

TRATTO CD $0 \le z \le L$

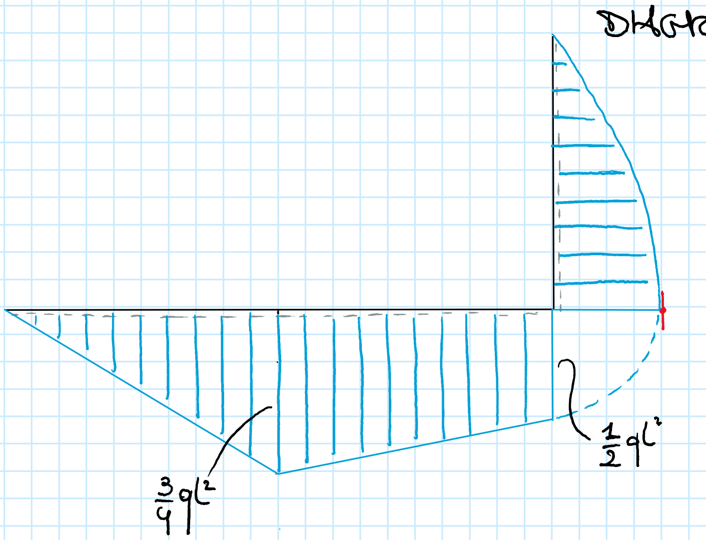


$$M^{(0)}(z) = qL(L-z) - \frac{q(L-z)^2}{2} =$$

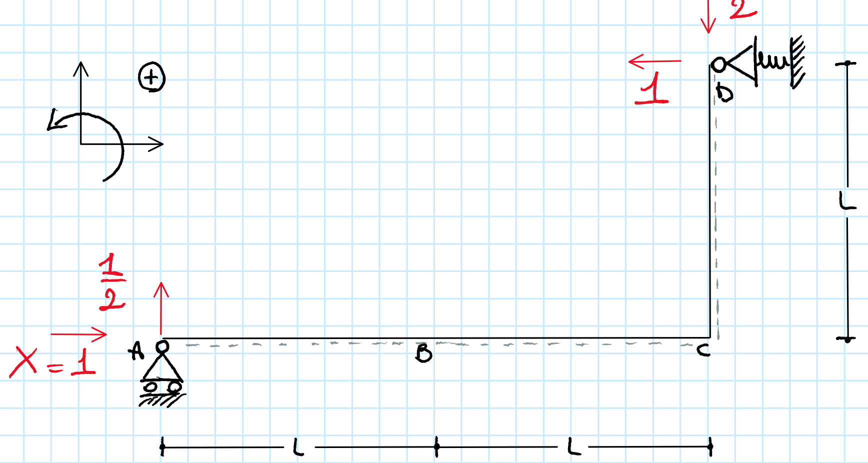
$$= \boxed{\frac{qL^2}{2} - \frac{qz^2}{2}}$$

$$\begin{cases} M_C = \frac{qL^2}{2} \\ M_D = \emptyset \end{cases}$$

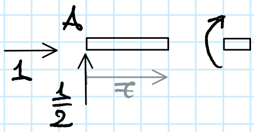
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $X = 1$

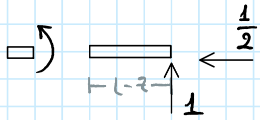


TRATTO AC $\quad 0 \le z \le 2L$



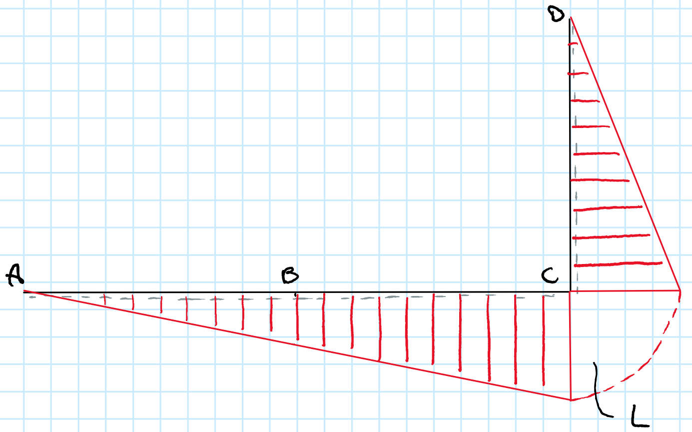
$$\boxed{M^{(1)}(z) = \frac{z}{2}} \qquad \begin{cases} M_A = 0 \\ M_C = L \end{cases}$$

TRATTO CD $\quad 0 \le z \le L$



$$\boxed{M^{(1)}(z) = L - z} \qquad \begin{cases} M_C = L \\ M_D = \phi \end{cases}$$

DIAGRAMMA $M^{(1)}(z)$



$$\alpha_{ve} = 1 \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \cdot \eta_j^{(r)} =$$

$$= \underbrace{R_{y_A}^{(1)} \cdot \eta_A^{0}}_{-\frac{\eta_A^0}{2}} + \underbrace{R_{x_D}^{(1)}}_{-1} \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon \underbrace{R_{xD}^{(r)}}_{R_{xD}^{(0)} + R_{xD}^{(1)} X}}$$

$$= \underbrace{-\frac{\eta_A^0}{2} - \varepsilon\left[qL + X\right]}$$

$$\underbrace{\alpha_{Vi}} = \int_{Str} M^{(1)} \underbrace{\frac{M^{(r)}}{EI}} + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$\to M^{(r)} = M^{(0)} + M^{(1)} X$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI}\, dStr + X \int_{Str} \frac{[M^{(1)}]^2}{EI}\, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$= \frac{1}{EI}\left\{ \int_{AB} \overbrace{\frac{z}{2}\left[\frac{3}{4} qLz\right]}^{\frac{3}{8}qLz^2} dz + \int_{BC} \overbrace{\frac{z}{2}\left[-\frac{qL}{4}z + qL^2\right]}^{-\frac{qL}{8}z^2+\frac{qL^2}{2}z} dz + \int_{CD} \overbrace{(L-z)\left[\frac{qL^2}{2} - \frac{q}{2}z^2\right]}^{\frac{qL^3}{2}-\frac{qL}{2}z^2-\frac{qL^2}{2}z+\frac{q}{2}z^3} dz \right\} +$$

$$+ \frac{X}{EI}\left\{\int_{AB}\frac{z^2}{4}\,dz + \int_{CD} \underbrace{(L-z)^2}_{L^2+z^2-2Lz}\, dz\right\} + \frac{\alpha\Delta T}{h}\int_{AB}\frac{z}{2}\,dz =$$

$$= \frac{1}{EI}\left\{\int_0^L \frac{3}{8}qLz^2 + \int_L^{2L}\left[-\frac{qL}{8}z^2+\frac{qL^2}{2}z\right]dz + \int_0^L\left[\frac{qL^3}{2}-\frac{qL}{2}z^2-\frac{qL^2}{2}z+\frac{q}{2}z^3\right]dz\right\}+$$

$$+\frac{X}{EI}\left\{\int_0^{2L}\frac{z^2}{4}\,dz + \int_0^L\left[L^2+z^2-2Lz\right]dz\right\} + \frac{\alpha\Delta T}{h}\int_0^L \frac{z}{2}\,dz =$$

$$= \frac{1}{EI}\left\{\frac{3}{8}qL\left[\frac{z^3}{3}\right]_0^L - \frac{qL}{8}\left[\frac{z^3}{3}\right]_L^{2L} + \frac{qL^2}{2}\left[\frac{z^2}{2}\right]_L^{2L} + \frac{qL^3}{2}\left[z\right]_0^L - \frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L +\right.$$

$$\left.-\frac{qL^2}{2}\left[\frac{z^2}{2}\right]_0^L + \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L\right\} + \frac{X}{EI}\left\{\frac{1}{4}\left[\frac{z^3}{3}\right]_0^{2L} + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right\}+$$

$$+\frac{\alpha\Delta T}{2h}\left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{qL^4}{EI}\left[\frac{1}{8} - \frac{7}{24} + \frac{3}{4} + \frac{1}{2} - \frac{1}{6} - \frac{1}{4} + \frac{1}{8}\right] + \frac{XL^3}{EI}\left[\frac{4}{12}+\frac{1}{3}\right] + \frac{\alpha\Delta T L^2}{4h} =$$

$$= \underbrace{\frac{19}{24}\frac{qL^4}{EI} + \frac{XL^3}{EI} + \frac{\alpha\Delta T}{4h}L^2}$$

$\alpha_{ve} = \alpha_{vi}$ fornisce

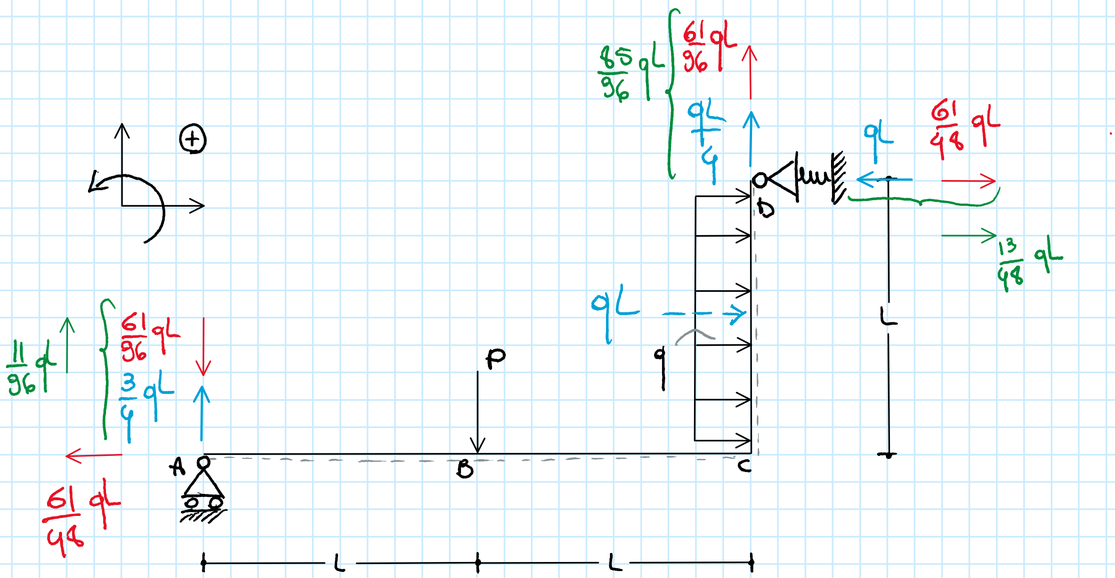
$$-\frac{\eta^{\circ}_{1}}{2} - \varepsilon q l - \varepsilon X = \frac{19}{24}\frac{ql^4}{EI} + X\frac{l^3}{EI} + \frac{\alpha \overline{\Delta T} l^2}{4h}$$

$$X\left[\frac{l^3}{EI} + \varepsilon\right] = -\left\{\frac{\eta^{\circ}_{1}}{2} + \varepsilon q l + \frac{19}{24}\frac{ql^4}{EI} + \frac{\alpha \overline{\Delta T}}{4h}\right\}$$

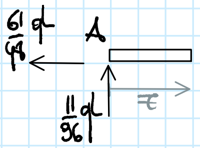
$$X\left[\frac{l^3}{EI} + \frac{l^3}{EI}\right] = -\left[\frac{ql^4}{2EI} + \frac{ql^4}{EI} + \frac{19}{24}\frac{ql^4}{EI} + \frac{ql^4}{4EI}\right]$$

$X = -\frac{61}{48} q l$ Negativo, verso contrario

SOLUZIONE SISTEMA ISOSTATICO PRINCIPALE



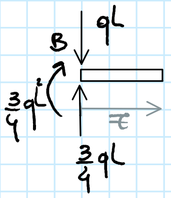
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(r)}(z) = \frac{11}{96} qLz}$$

$$\begin{cases} M_A = 0 \\ M_B = \dfrac{11}{96} qL^2 \end{cases}$$

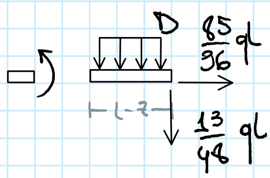
TRATTO BC $L \le z \le 2L$



$$\boxed{M^{(r)}(z) = \frac{11}{96} qLz - qL(z-L)}$$

$$\begin{cases} M_B = \dfrac{11}{96} qL^2 \\ M_C = -\dfrac{37}{48} qL^2 \end{cases}$$

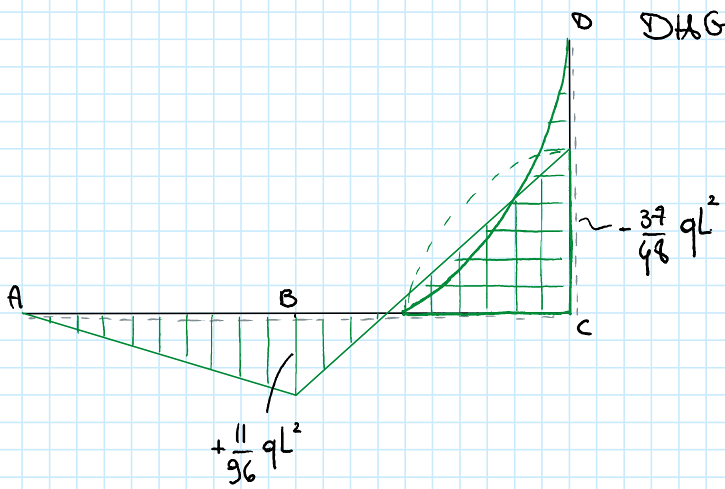
TRATTO CD $0 \le z \le L$



$$\boxed{M^{(r)}(z) = -\frac{13}{48} qL(L-z) - \frac{q(L-z)^2}{2}}$$

$$\begin{cases} M_C = -\dfrac{37}{48} qL^2 \\ M_D = \emptyset \end{cases}$$

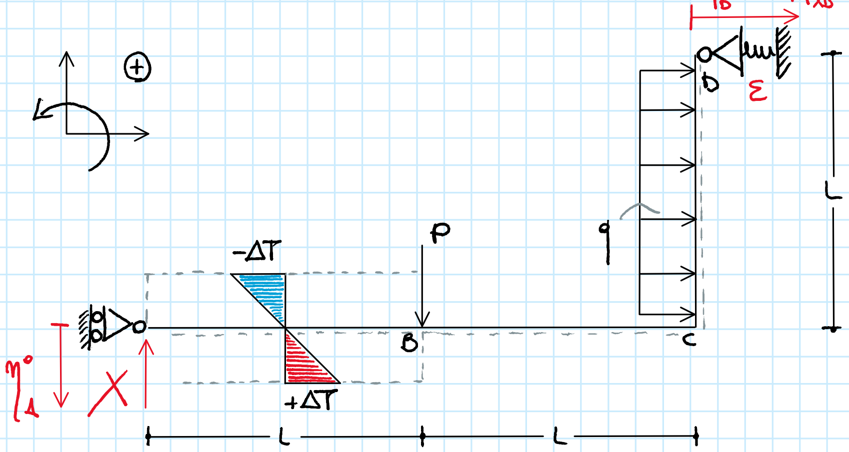
DIAGRAMMA $M^{(r)}(z)$



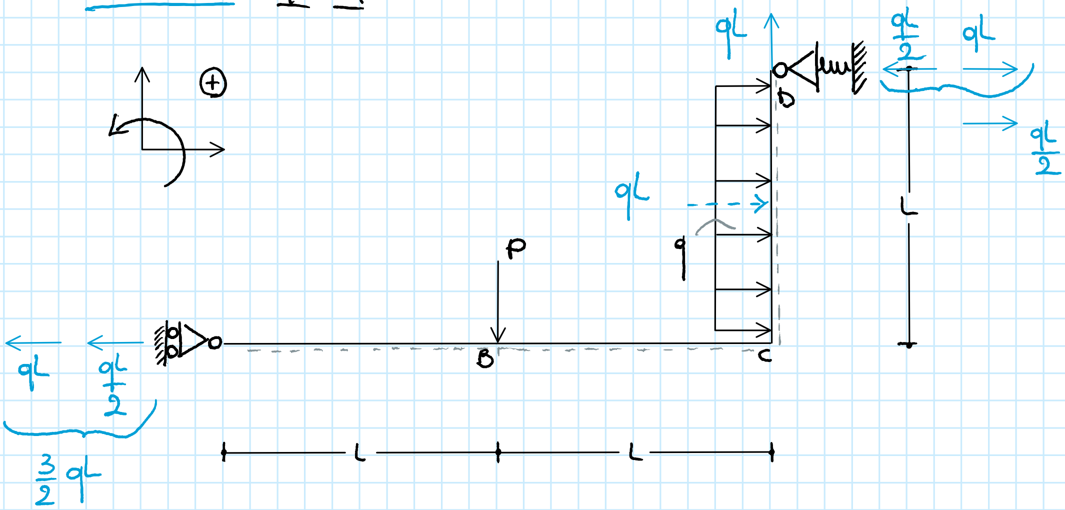
SOLUZIONE # 2

- SISTEMA ISOSTATICO PRINCIPALE

$$\eta_D = -\varepsilon R_{x_D}^{(1)} = -\varepsilon\left[R_{x_D}^{(0)} + X R_{x_D}^{(1)}\right]$$



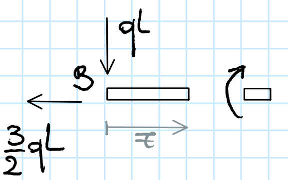
SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO AB $0 \le z \le L$

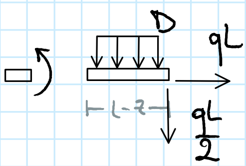
$$\boxed{M^{(0)}(z) = \emptyset}$$

TRATTO BC $L \le z \le 2L$



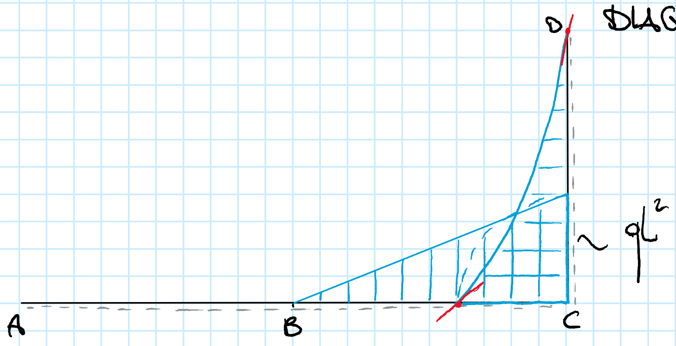
$$\boxed{M^{(0)}(z) = -qL(z-L)} \qquad \begin{cases} M_B = 0 \\ M_C = -qL^2 \end{cases}$$

TRATTO CD

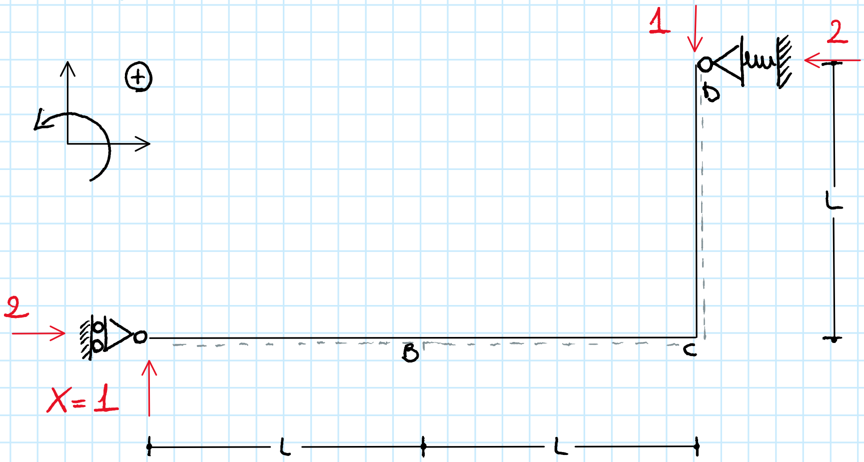


$$\boxed{M^{(0)}(z) = -\frac{qL}{2}(L-z) - \frac{q(L-z)^2}{2}} \qquad \begin{cases} M_C = -qL^2 \\ M_D = \emptyset \end{cases}$$

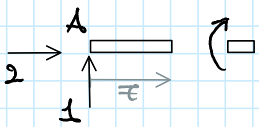
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $X=1$



TRATTO AB $0 \le z \le L$



$\boxed{M^{(1)}(z) = z}$

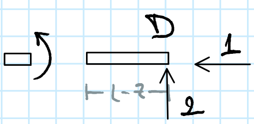
$$\begin{cases} M_A = 0 \\ M_B = L \end{cases}$$

TRATTO BC $L \le z \le 2L$

Come tratto BC $\boxed{M^{(1)}(z) = z}$

$$\begin{cases} M_B = L \\ M_C = 2L \end{cases}$$

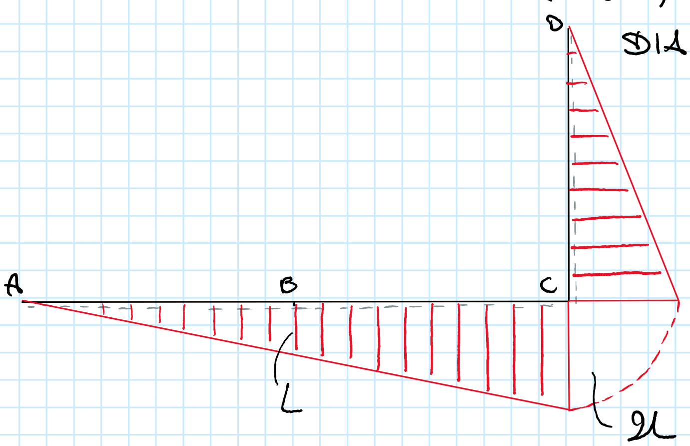
TRATTO CD $0 \le z \le L$



$\boxed{M^{(1)}(z) = 2(L-z)}$

$$\begin{cases} M_C = 2L \\ M_D = \emptyset \end{cases}$$

DIAGRAMMA $M^{(1)}(z)$



$$\mathcal{L}_{ve} = 1\cdot\left(-\eta_A^{\circ}\right) + \sum_j R_j^{(1)}\,\eta_j^{(r)} = -\eta_A^{\circ} + \underbrace{R_{X_D}^{(1)}}_{-2}\cdot\underbrace{\eta_D^{(r)}}_{-\varepsilon\left\{\underbrace{R_{X_D}^{(0)}}_{\frac{qL}{2}} + \underbrace{R_{X_D}^{(1)}}_{-2} X\right\}} =$$

$$= \underbrace{-\eta_A^{\circ} + 2\varepsilon\left[\frac{qL}{2} - 2X\right]}$$

$$\mathcal{L}_{vi} = \int_{Str} M^{(1)} M^{(r)}\, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$= \frac{1}{EI}\left\{\int_{Str} M^{(1)} M^{(0)}\, dStr\right\} + \frac{X}{EI}\left\{\int_{Str}\left[M^{(1)}\right]^2 dStr\right\} + \int_{Str} M^{(1)} \frac{\alpha\Delta T}{h}\, dStr =$$

$$= \frac{1}{EI}\left\{\int_{BC} M^{(1)} M^{(0)}\, dz + \int_{CD} M^{(1)} M^{(0)}\, dz\right\} + \frac{X}{EI}\left\{\int_{AC}\left[M^{(1)}\right]^2 dz + \int_{CD}\left[M^{(1)}\right]^2 dz\right\} + \frac{\alpha\Delta T}{h}\int_{AB} M^{(1)}\, dz =$$

$$= \frac{1}{EI}\left\{\int_L^{2L} \overbrace{z\left[-qL(z-L)\right]}^{-qL(z^2-Lz)} dz + \int_0^L \overbrace{2(L-z)\left[-\frac{qL}{2}(L-z) - \frac{q(L-z)^2}{2}\right]}^{(L-z)\left[-qL(L-z) - q(L^2+z^2-2Lz)\right]} dz\right\} +$$

$$+ \frac{X}{EI}\left\{\int_0^{2L} z^2\, dz + \int_0^L 4\underbrace{(L-z)^2}_{L^2+z^2-2Lz} dz\right\} + \frac{\alpha\Delta T}{h}\int_0^L z\, dz =$$

$$= \frac{1}{EI}\left\{-qL\left[\frac{z^3}{3}\right]_L^{2L} + qL^2\left[\frac{z^2}{2}\right]_L^{2L} - 2qL^3\left[z\right]_0^L - qL\left[\frac{z^3}{3}\right]_0^L + 3qL\left[\frac{z^2}{2}\right]_0^L +\right.$$

$$\left. + 2qL^2\left[\frac{z^2}{2}\right]_0^L + q\left[\frac{z^4}{4}\right]_0^L - 3qL\left[\frac{z^3}{3}\right]_0^L\right\} + \frac{X}{EI}\left\{\left[\frac{z^3}{3}\right]_0^{2L} + 4L^2\left[z\right]_0^L +\right.$$

$$\left. + 4\left[\frac{z^3}{3}\right]_0^L - 8L\left[\frac{z^2}{2}\right]_0^L\right\} + \frac{\alpha\Delta T}{h}\left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{1}{EI}\left\{-\frac{7}{3}qL^4 + \frac{3}{2}qL^4 - 2qL^4 - qL\frac{L^3}{3} + 3qL^2\frac{L^2}{2} + 2qL^2\frac{L^2}{2} + \frac{qL^4}{4} - 3qL\frac{L^3}{3}\right\} +$$

$$+ \frac{X}{EI}\left\{\frac{1}{3}8L^3 + 4L^2\cdot L + 4\frac{L^3}{3} - 8L\frac{L^2}{2}\right\} + \frac{\alpha\Delta T}{h}\frac{L^2}{2}$$

$$= \underbrace{-\frac{17}{12}\frac{qL^4}{EI} + \frac{4L^3}{EI}X + \frac{\alpha\Delta T}{h}\frac{L^2}{2}}$$

$\mathcal{L}_{ve} = \mathcal{L}_{vi}$ fornisce

$$-\eta_A^{\circ} + 2\varepsilon\left[\frac{qL}{2} - 2X\right] = -\frac{5}{12}\frac{qL^4}{EI} + \frac{4L^3}{EI}X + \frac{\alpha\Delta T}{h}\frac{L^2}{2}$$

$$-\frac{qL^4}{EI} + \frac{qL^4}{EI} - \frac{4L^3}{EI}X = -\frac{17}{12}\frac{qL^4}{EI} + \frac{4L^3}{EI}X + \frac{qL^4}{2EI}$$

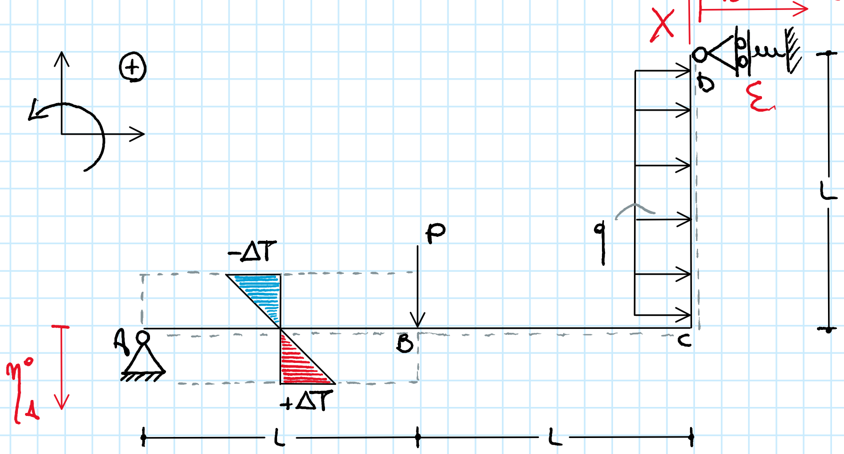
$$-8X = qL\left(\frac{1}{2} - \frac{17}{12}\right)$$

$$X = +\frac{11}{96}qL \quad \text{POSITIVO}$$

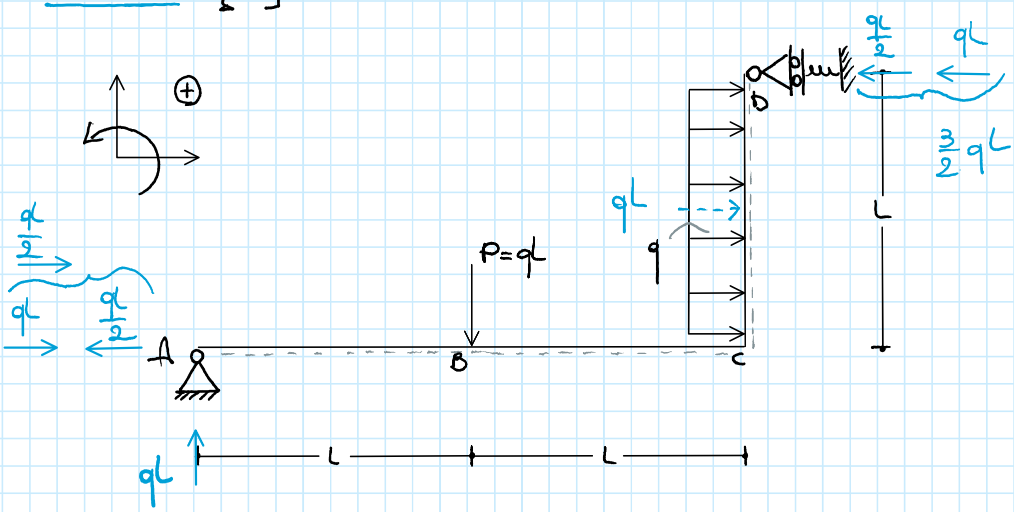
SOLUZIONE # 3

- SISTEMA ISOSTATICO PRINCIPALE

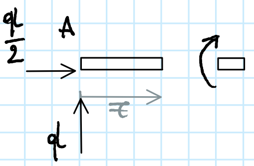
$$\eta_D = -\varepsilon R^{(1)}_{XD} = -\varepsilon\left[R^{(0)}_{XD} + X R^{(1)}_{XD}\right]$$



SCHEMA [0] SOLO CARICHI ESTERNI



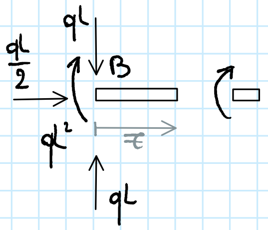
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qLz}$$

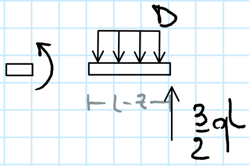
$$\begin{cases} M_A = \emptyset \\ M_B = qL^2 \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL^2}$$ cost.

TRATTO CD $\quad 0 \le z \le L$

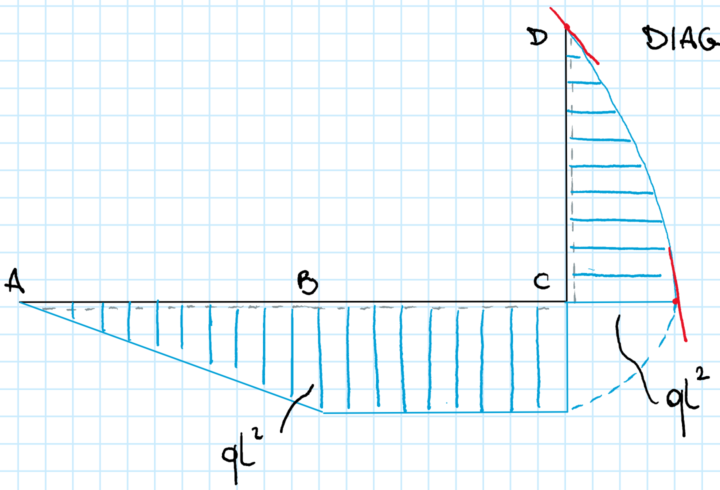


$$M^{(0)}(z) = \frac{3}{2}qL(L-z) - \frac{q}{2}(L-z)^2$$

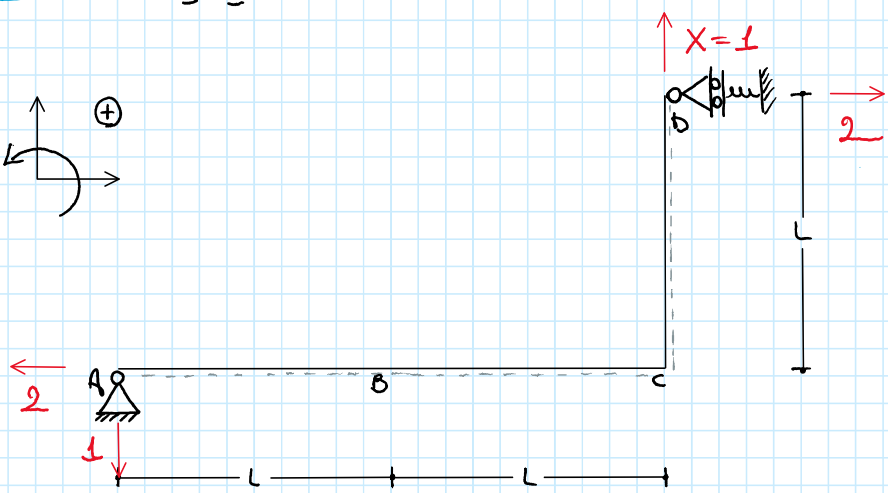
$$= \frac{3}{2}qL^2 - \frac{3}{2}qLz - \frac{1}{2}qL^2 - \frac{1}{2}qz^2 + qLz$$

$$= \boxed{qL^2 - \frac{1}{2}qz^2 - \frac{1}{2}qLz} \qquad \begin{cases} M_C = qL^2 \\ M_D = \emptyset \end{cases}$$

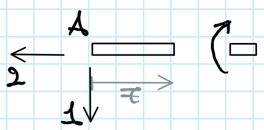
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $X = 1$

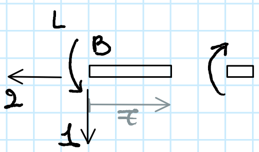


TRATTO AB $\quad 0 \le z \le L$



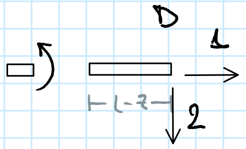
$$\boxed{M^{(1)}(z) = -z} \qquad \begin{cases} M_A = 0 \\ M_B = -L \end{cases}$$

TRATTO BC $\quad 0 \le z \le L$



$$\boxed{M^{(1)}(z) = -L - z} \qquad \begin{cases} M_B = -L \\ M_C = -2L \end{cases}$$

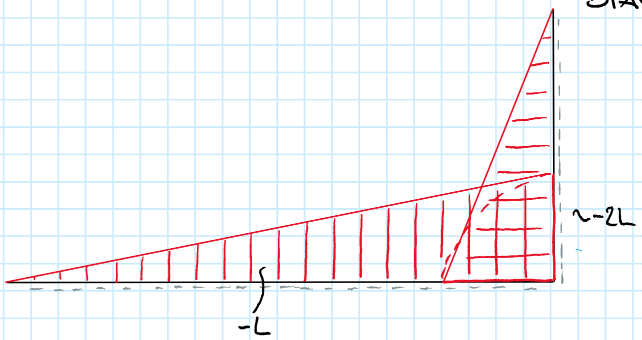
TRATTO CD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -2(L-z)}$$

$$\begin{cases} M_B = -2L \\ M_C = \emptyset \end{cases}$$

DIAGRAMMA $M^{(1)}(z)$



$$\alpha_{ve} = 1 \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \cdot \eta_j^{(r)} =$$

$$= \underbrace{R_{y_A}^{(1)}}_{-1} \cdot \underbrace{\eta_A^{o}}_{-\eta_A^{o}} + \underbrace{R_{X_D}^{(1)}}_{+2} \cdot \underbrace{\eta_D^{(H)}}_{-\varepsilon R_{X_D}^{(r)}}$$

$$\underbrace{= +\eta_A^{o} - 2\varepsilon\left[-\frac{3}{2}qL + 2X\right]}$$

$$R_{X_D}^{(0)} + R_{X_D}^{(1)} X$$

$$-\frac{3}{2}qL + 2X$$

$$\alpha_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + X \int_{Str} \frac{[M^{(1)}]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha\Delta T}{h} dStr =$$

$$= \frac{1}{EI}\left\{ \int_{AB} -z(qLz)\,dz + \int_{BC} \overbrace{-(L+z)\,qL^2}^{-qL^3 - qL^2 z}\,dz + \int_{CD} \overbrace{\underbrace{-2(L-z)}_{-2L+2z}\left(qL^2 - \frac{1}{2}qz^2 - \frac{1}{2}qLz\right)}^{-2qL^3 + qLz^2 + qL^2 z + 2qL^2 z - qz^3 - qLz^2} dz \right\} +$$

$$+ \frac{X}{EI}\left\{ \int_{AB} (-z)^2 dz + \int_{BC} [-(L+z)]^2 dz + \int_{CD} \underbrace{[-2(L-z)]^2}_{4L^2+4z^2-8Lz} dz \right\} + \frac{\alpha\Delta T}{h}\int_{AB} (-z)\,dz =$$

$$= \frac{1}{EI}\left\{ -qL\left[\frac{z^3}{3}\right]_0^L - qL^3\left[z\right]_0^L - qL^2\left[\frac{z^2}{2}\right]_0^L - 2qL^3\left[z\right]_0^L + qL\left[\frac{z^3}{3}\right]_0^L + qL^2\left[\frac{z^2}{2}\right]_0^L + \right.$$

$$+2qL^2\left[\frac{z^2}{2}\right]_0^L - q\left[\frac{z^4}{4}\right]_0^L - qL\left[\frac{z^3}{3}\right]_0^L\Bigg\} + \frac{X}{EI}\Bigg\{\left[\frac{z^3}{3}\right]_0^L + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L + 2L\left[\frac{z^2}{2}\right]_0^L +$$

$$+4L^2\left[z\right]_0^L + 4\left[\frac{z^3}{3}\right]_0^L - 4L\left[\frac{z^2}{2}\right]_0^L\Bigg\} - \frac{\alpha\Delta T}{h}\left[\frac{z^2}{2}\right] =$$

$$= \frac{1}{EI}\Bigg\{-\frac{qL^4}{3} - qL^4 - \frac{qL^4}{2} - 2qL^4 + \frac{qL^4}{3} + \frac{qL^4}{2} + qL^4 - \frac{qL^4}{4} - \frac{qL^4}{3}\Bigg\} +$$

$$+\frac{X}{EI}\Bigg\{\frac{L^3}{3} + L^3 + \frac{L^3}{3} + L^3 + 4L^3 + \frac{4}{3}L^3 - 4L^3\Bigg\} - \frac{\alpha\Delta T}{h}\cdot\frac{L^2}{2} =$$

$$= \underbrace{-\frac{31}{12}\frac{qL^4}{EI} + 4\frac{XL^3}{EI} - \frac{L^2}{2}\frac{\alpha\Delta T}{h}}$$

$\mathcal{L}_{ve} = \mathcal{L}_{vi}$ fornisce

$$+\eta_A^0 - 2\varepsilon\left[-\frac{3}{2}qL + 2X\right] = -\frac{31}{12}\frac{qL^4}{EI} + 4\frac{XL^3}{EI} - \frac{L^2}{2}\frac{\alpha\Delta T}{h}$$

$$+\frac{qL^4}{EI} + \frac{L^3}{EI}3qL - \frac{XL^3}{EI}4 = -\frac{31}{12}\frac{qL^4}{EI} + 4\frac{XL^3}{EI} - \frac{1}{2}\frac{qL^4}{EI}$$

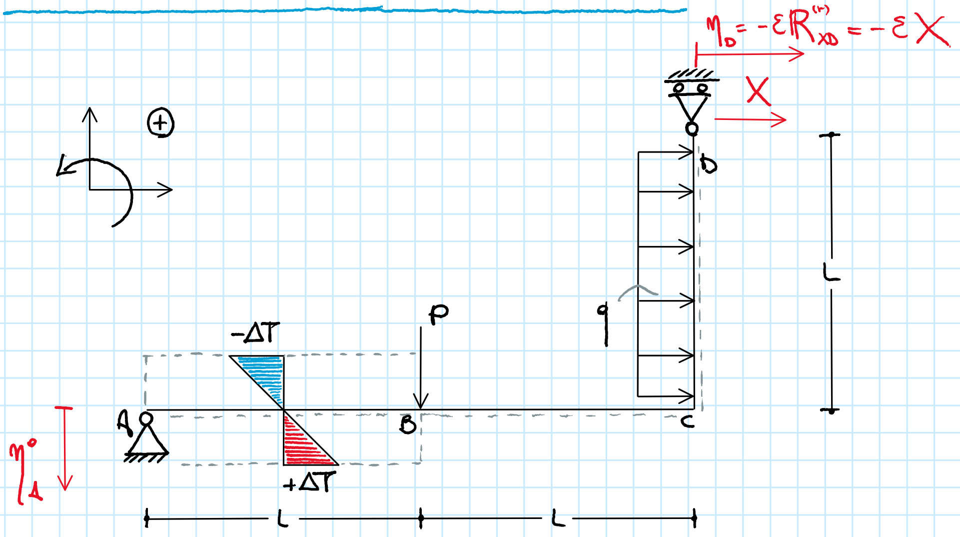
$$+8\frac{XL^3}{EI} = \frac{qL^4}{EI}\left[+1+3+\frac{31}{12}+\frac{1}{2}\right]$$

$$+8\frac{XL^3}{EI} = \frac{qL^4}{EI}\left[\frac{85}{12}\right]$$

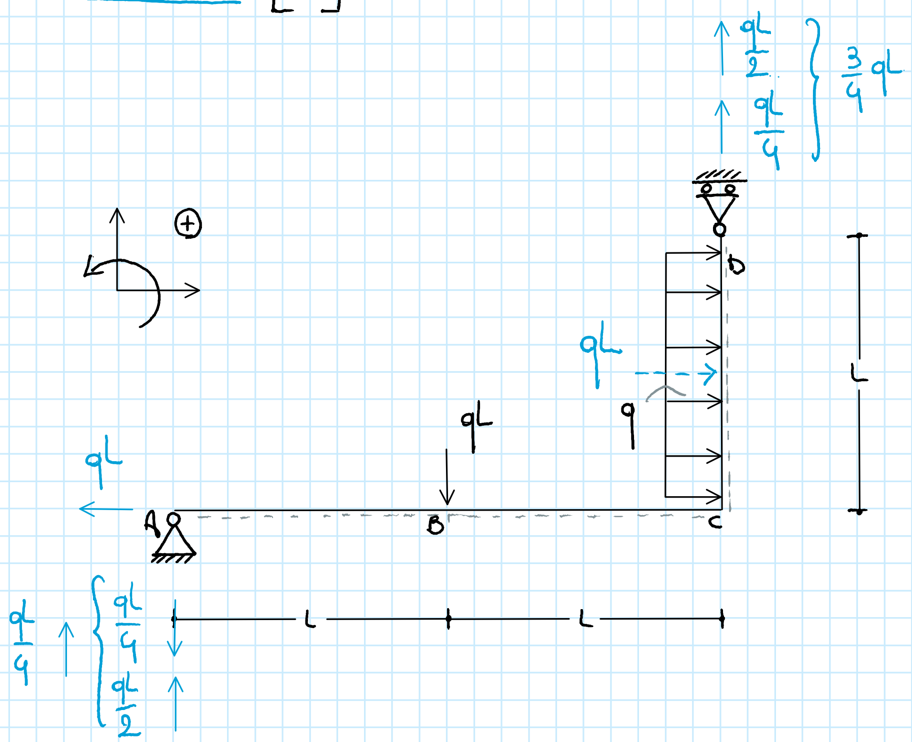
$X = \frac{85}{96}qL$ Positivo, verso ok!

SOLUZIONE # 4

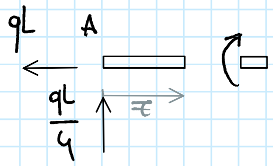
• SISTEMA ISOSTATICO PRINCIPALE



SCHEMA [0] SOLO CARICHI ESTERNI



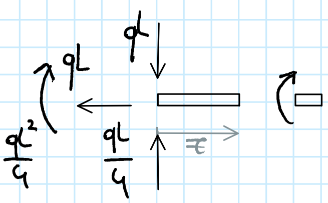
TRATTO AB $0 \leq z \leq L$



$$\boxed{M^{(0)}(z) = \frac{qL}{4} z}$$

$$\begin{cases} M_A = 0 \\ M_B = \frac{qL^2}{4} \end{cases}$$

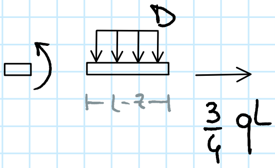
TRATTO BC $0 \leq z \leq L$



$$\boxed{M^{(0)}(z) = \frac{qL^2}{4} - \frac{3}{4} qLz}$$

$$\begin{cases} M_B = \frac{qL^2}{4} \\ M_C = -\frac{1}{2} qL^2 \end{cases}$$

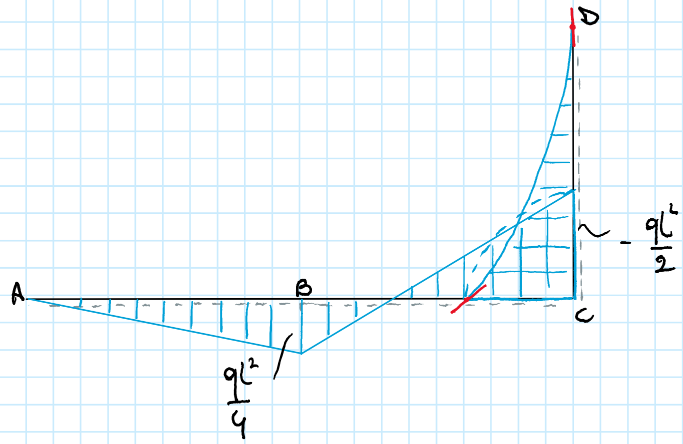
TRATTO CD $0 \le z \le L$



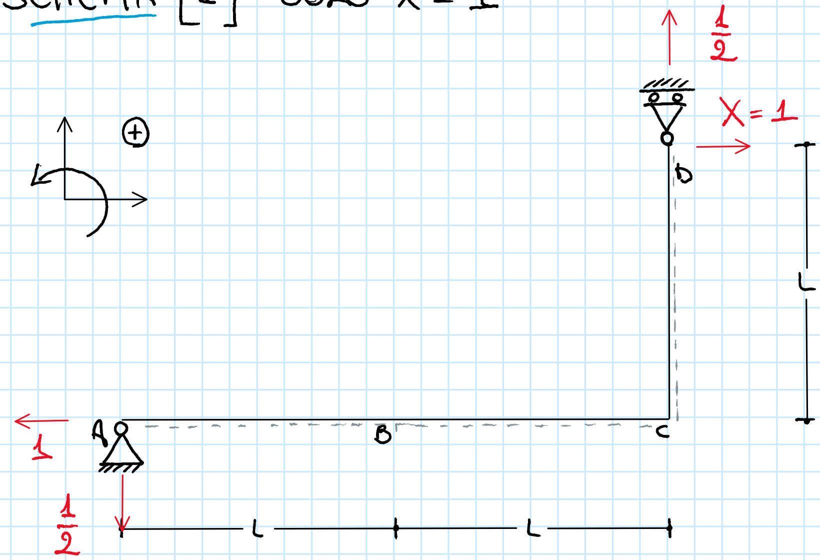
$$\boxed{M^{(0)}(z) = -\frac{q}{2}(L-z)^2}$$

$$\begin{cases} M_C = -\frac{1}{2} qL^2 \\ M_D = 0 \end{cases}$$

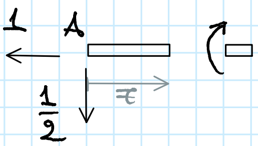
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $X = 1$



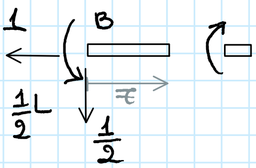
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{1}{2} z}$$

$$\begin{cases} M_A = \emptyset \\ M_B = -\frac{1}{2} L \end{cases}$$

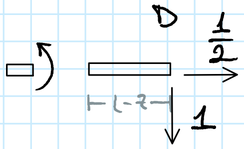
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{1}{2}(L+z)}$$

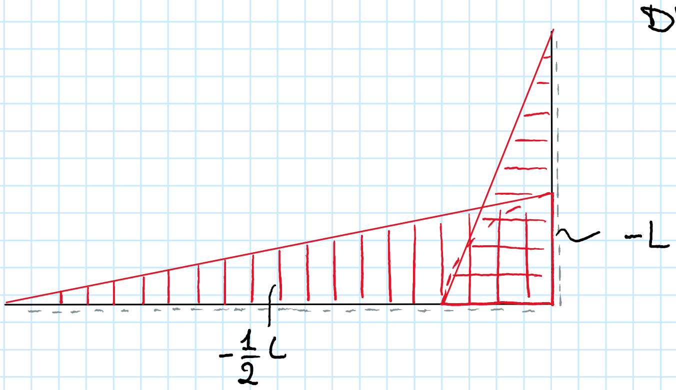
$$\begin{cases} M_B = -\frac{1}{2} L \\ M_C = -L \end{cases}$$

TRATTO CD $\quad 0 \le z \le L$



$$\boxed{M^{(1)}(z) = -(L-z)} \qquad \begin{cases} M_C = -L \\ M_D = \emptyset \end{cases}$$

DIAGRAMMA $M^{(1)}(z)$



$$\underset{\sim}{L_{ve}} = 1 \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \cdot \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon X} + \underbrace{R_{y_A}^{(1)}}_{-\frac{1}{2}} \cdot \underbrace{\eta_A^{\circ}}_{-\eta_A^{\circ}}$$

$$= \underbrace{-\varepsilon X + \frac{1}{2}\eta_A^{\circ}}$$

$$\underset{\sim}{L_{vi}} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI}\, dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI}\, dStr + X \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI}\, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$-\frac{qL^3}{8} - \frac{qL^2}{8}z + \frac{3}{8}qL^2 z + \frac{3}{8}qLz^2$$

$$= \frac{1}{EI}\left\{ \int_{AB} -\frac{1}{2}z\left[\frac{qL}{4}z\right] dz + \int_{BC} -\frac{1}{2}(L+z)\left[\frac{qL^2}{4} - \frac{3}{4}qLz\right] dz + \int_{CD} +(L-z)\,\frac{q}{2}(L-z)^2\, dz \right\} +$$

$$\frac{q}{2}(L-z)^3 \qquad L^3 - z^3 - 3L^2 z + 3Lz^2$$

$$+ \frac{X}{EI}\left\{ \int_{AB} \left(-\frac{1}{2}z\right)^2 dz + \int_{BC} \left[-\frac{1}{2}(L+z)\right]^2 dz + \int_{CD} \left[-(L-z)\right]^2 + \frac{\alpha \Delta T}{h} \int_{AB} -\frac{1}{2}z\, dz \right.$$

$$+\frac{L^2}{4} + \frac{z^2}{4} + \frac{zL}{2} \qquad L^2 + z^2 - 2Lz$$

$$= \frac{1}{EI}\left\{ -\frac{qL}{8}\left[\frac{z^3}{3}\right]_0^L - \frac{qL^3}{8}\left[z\right]_0^L - \frac{qL^2}{8}\left[\frac{z^2}{2}\right]_0^L + \frac{3}{8}qL^2\left[\frac{z^2}{2}\right]_0^L + \frac{3}{8}qL\left[\frac{z^3}{3}\right]_0^L + \frac{qL^3}{2}\left[z\right]_0^L - \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L + \right.$$

$$\left. - \frac{3}{2}qL^2\left[\frac{z^2}{2}\right]_0^L + \frac{3}{2}qL\left[\frac{z^3}{3}\right]_0^L \right\} + \frac{X}{EI}\left\{ +\frac{1}{4}\left[\frac{z^3}{3}\right]_0^L + \frac{L^2}{4}\left[z\right]_0^L + \frac{1}{4}\left[\frac{z^3}{3}\right]_0^L + \frac{L}{2}\left[\frac{z^2}{2}\right]_0^L + \right.$$

$$+L^2\left[z\right]_0^L+\left[\frac{z^3}{3}\right]_0^L-2L\left[\frac{z^2}{2}\right]_0^L\Bigg\}-\frac{\alpha\Delta T}{h}\,\frac{1}{2}\left[\frac{z^2}{2}\right]_0^L=$$

$$=\frac{1}{EI}\left\{-\frac{1}{24}qL^4-\cancel{\frac{1}{8}}qL^4-\frac{1}{16}qL^4+\frac{3}{16}qL^4+\cancel{\frac{1}{8}}qL^4+\frac{1}{2}qL^4-\frac{1}{8}qL^4-\frac{3}{4}qL^4+\frac{1}{2}qL^4\right\}+$$

$$+\frac{X}{EI}\left\{\frac{L^3}{12}+\frac{L^3}{4}+\frac{L^3}{12}+\frac{L^3}{4}+L^3+\frac{L^3}{3}-L^3\right\}-\frac{\alpha\Delta T}{h}\,\frac{L^2}{4}=$$

$$=\underbrace{+\frac{10}{48}\frac{qL^4}{EI}+\frac{XL^3}{EI}-\frac{L^2}{4}\,\frac{\alpha\Delta T}{h}}=$$

$\mathcal{L}_{ve}=\mathcal{L}_{vi}$ fornisce

$$-\varepsilon X+\frac{1}{2}\varphi_A^0=\frac{10}{48}\frac{qL^4}{EI}+\frac{XL^3}{EI}-\frac{L^2}{4}\,\frac{\alpha\Delta T}{h}$$

$$-\frac{XL^3}{EI}+\frac{1}{2}\frac{qL^4}{EI}=\frac{10}{48}\frac{qL^4}{EI}+\frac{XL^3}{EI}-\frac{1}{4}\frac{qL^4}{EI}$$

$$2\frac{XL^3}{EI}=\frac{qL^4}{EI}\left[-\frac{10}{48}+\frac{1}{4}+\frac{1}{2}\right]$$

$$X=+\frac{\cancel{26}^{\,13}}{48}\cdot\frac{1}{\cancel{2}}qL$$

$X=+\frac{13}{48}qL$ Positivo, ok!