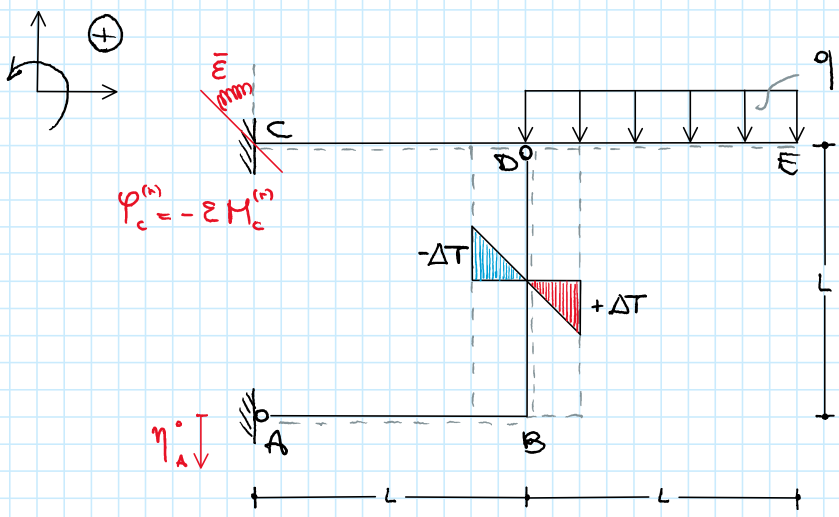
IPER 7

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA DETERMINANDO IL DIAGRAMMA DEL MOMENTO



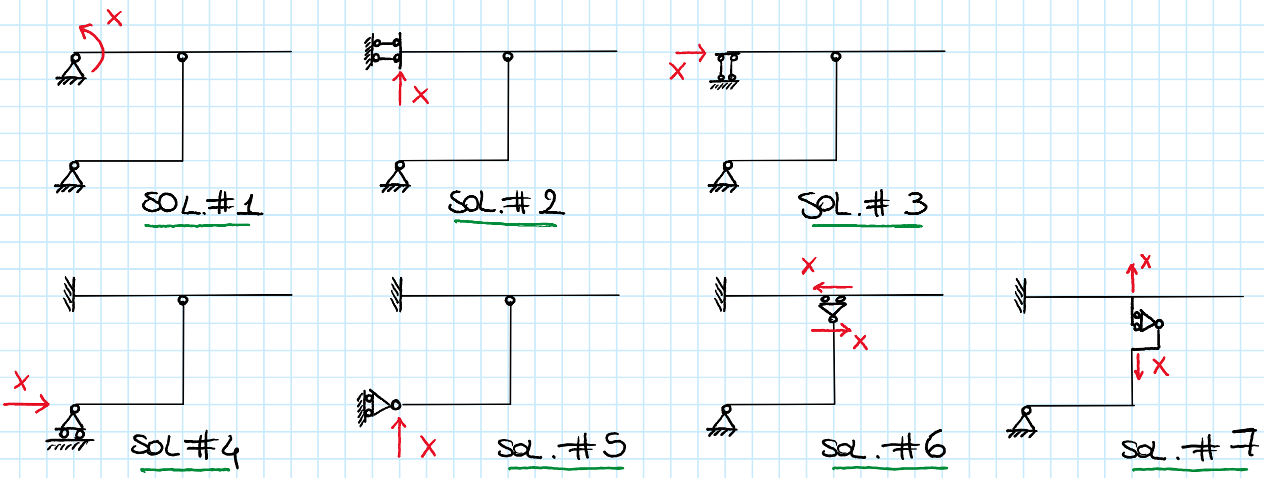
POSIZIONI

$$|\varepsilon| = \frac{L}{EI}$$

$$|\eta_A^\circ| = \frac{qL^4}{3EI}$$

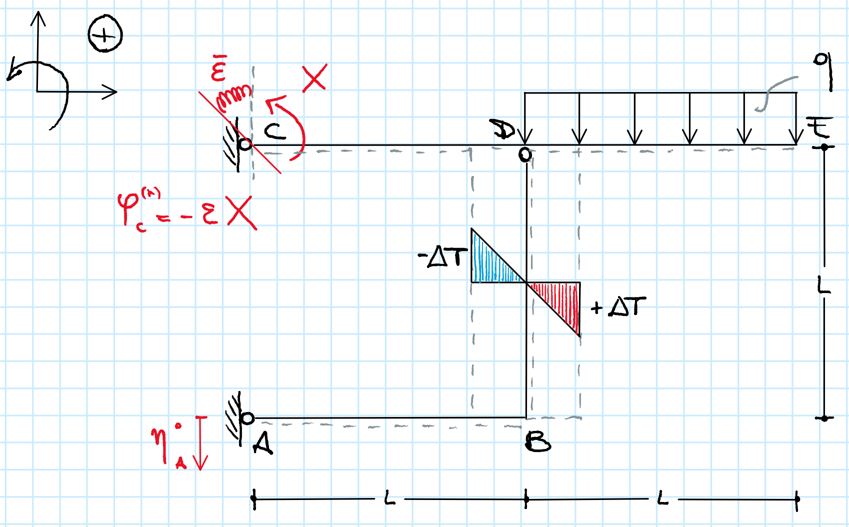
$$\left|\frac{\alpha \Delta T}{h}\right| = \frac{2qL^2}{EI}$$

SOLUZIONI POSSIBILI:

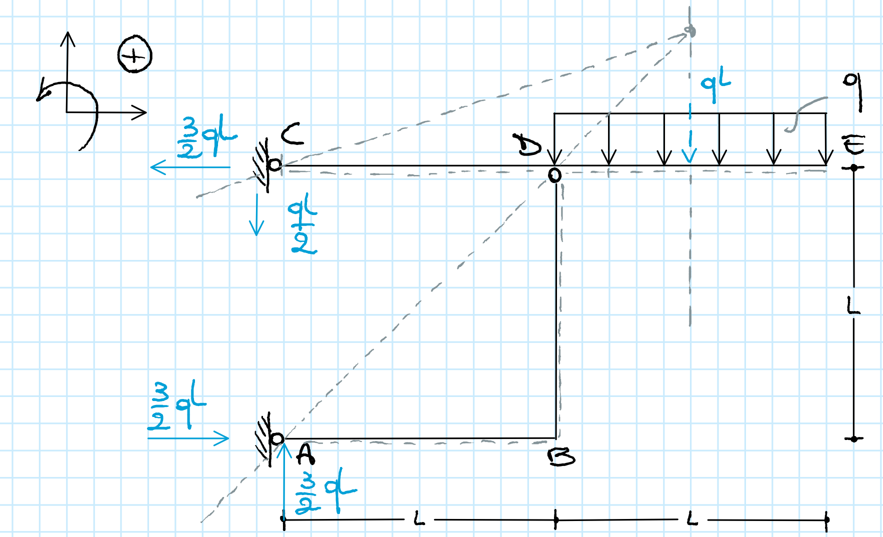


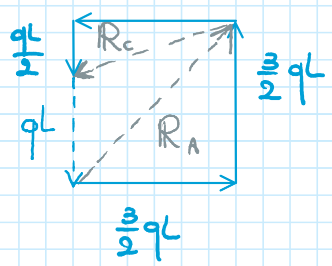
SOLUZIONE #1

- SISTEMA PRINCIPALE ISOSTATICO

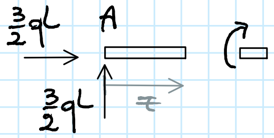


SCHEMA [0] SOLO CARICHI ESTERNI





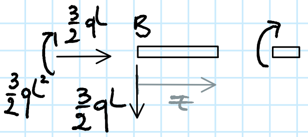
TRATTO AB $\quad 0 \leq z \leq L$



$$\boxed{M^{(0)}(z) = \frac{3}{2}qLz}$$

$$\begin{cases} M_A = \emptyset \\ M_B = \frac{3}{2}qL^2 \end{cases}$$

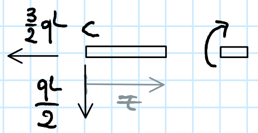
TRATTO BD $\quad 0 \leq z \leq L$



$$\boxed{M^{(0)}(z) = \frac{3}{2}qL^2 - \frac{3}{2}qLz}$$

$$\begin{cases} M_B = \frac{3}{2}qL^2 \\ M_D = \emptyset \end{cases}$$

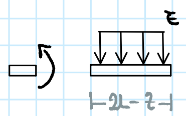
TRATTO CD $\quad 0 \leq z \leq L$



$$\boxed{M^{(0)}(z) = -\frac{qL}{2}z}$$

$$\begin{cases} M_C = \emptyset \\ M_D = -\frac{qL^2}{2} \end{cases}$$

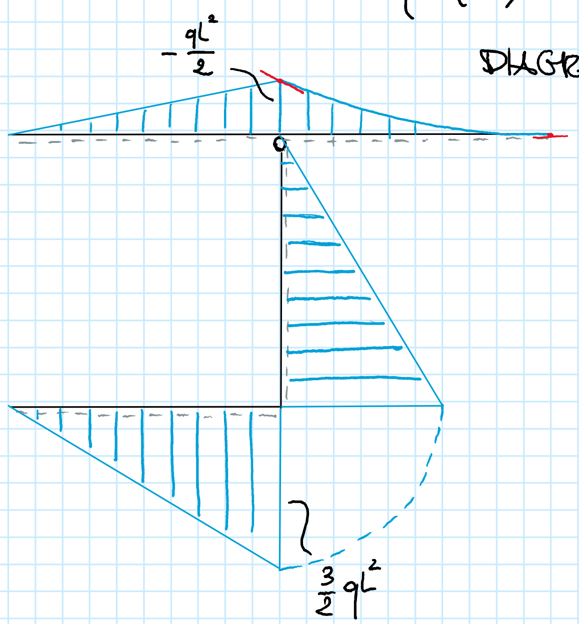
TRATTO DE $\quad L \leq z \leq 2L$



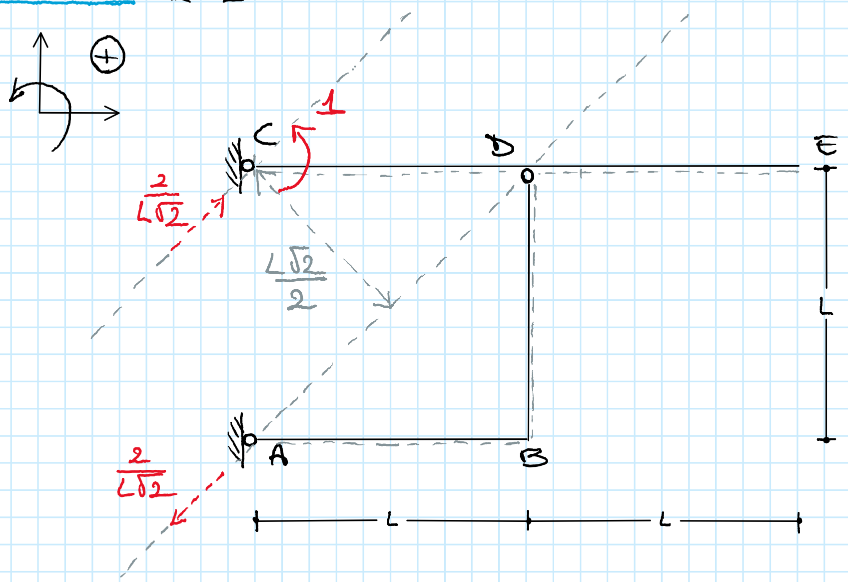
$$\boxed{M^{(0)}(z) = -\frac{q}{2}(2L-z)^2}$$

$$\begin{cases} M_D = -\frac{qL^2}{2} \\ M_E = \emptyset \end{cases}$$

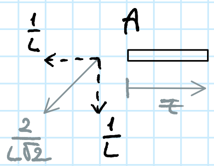
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $X=1$



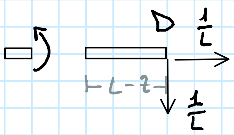
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -\frac{z}{L}}$$

$$\begin{cases} M_A = \emptyset \\ M_B = -1 \end{cases}$$

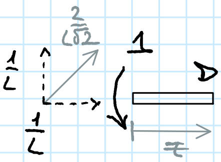
TRATTO BD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -\frac{1}{L}(L-z)}$$

$$\begin{cases} M_B = -1 \\ M_D = \emptyset \end{cases}$$

TRATTO CD $0 \le z \le L$

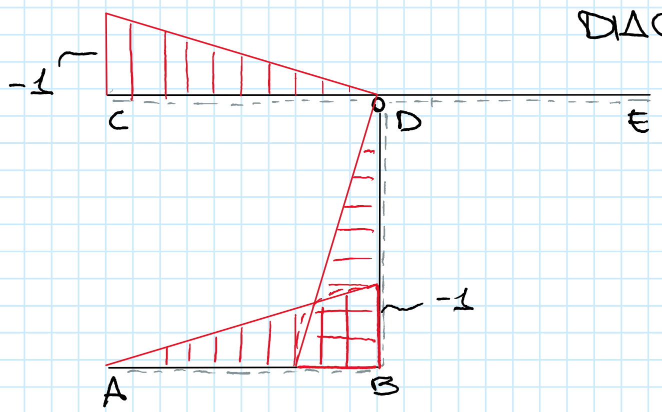


$$\boxed{M^{(1)}(z) = -1 + \frac{z}{L}}$$

$$\begin{cases} M_C = -1 \\ M_D = \emptyset \end{cases}$$

TRATTO DE Scarico

DIAGRAMMA $M^{(1)}(z)$



$$\underbrace{\mathcal{L}_{ve}} = X_i \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \cdot \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{\varphi_c^{(r)}}_{-\bar{\varepsilon}X} + R_{y_A}^{(1)} \cdot \eta_A^{0} = \underbrace{-\bar{\varepsilon} X + \frac{\eta_A^{0}}{L}}$$

$$\underbrace{\mathcal{L}_{vi}} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} \, dStr + \int_{Str} M^{(f)} \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} \, dStr + X \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} \, dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} \overbrace{\left(-\frac{z}{L}\right)\left(\frac{3}{2}qLz\right)}^{-\frac{3}{2}qz^2} dz + \int_{BD} \overbrace{-\frac{1}{\cancel{L}}(L-z)\left[\frac{3}{2}qL^{\cancel{2}} - \frac{3}{2}q\cancel{L}z\right]}^{\frac{3}{2}qLz - \frac{3}{2}qz^2 - \frac{3}{2}qL^2 + \frac{3}{2}qzL} dz + \int_{CD} \underbrace{\left(-1 + \frac{z}{L}\right)\left(-\frac{qL}{2}z\right)}_{\frac{qL}{2}z - \frac{q}{2}z^2} dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_{AB} \underbrace{\left(-\frac{z}{L}\right)^2}_{\frac{z^2}{L^2}} dz + \int_{BD} \underbrace{\left[-\frac{1}{L}(L-z)\right]^2}_{1 + \frac{z^2}{L^2} - \frac{2z}{L}} dz + \int_{CD} \underbrace{\left(\frac{z}{L} - 1\right)^2}_{\frac{z^2}{L^2} + 1 - \frac{2z}{L}} dz \right\} + \int_{BD} \left(\frac{z}{L} - 1\right)\left(\frac{\alpha \overline{\Delta T}}{h}\right) dz =$$

$$= \frac{1}{EI} \left\{ -\frac{3}{2}q\left[\frac{z^3}{3}\right]_0^L + \frac{3}{2}qL\left[\frac{z^2}{2}\right]_0^L - \frac{3}{2}q\left[\frac{z^3}{3}\right]_0^L - \frac{3}{2}qL^2\left[z\right]_0^L + \frac{3}{2}qL\left[\frac{z^2}{2}\right]_0^L + \right.$$

$$\left. + \frac{qL}{2}\left[\frac{z^2}{2}\right] - \frac{q}{2}\left[\frac{z^3}{3}\right] \right\} + \frac{X}{EI} \left\{ \frac{1}{L^2}\left[\frac{z^3}{3}\right]_0^L + \left[z\right]_0^L + \frac{1}{L^2}\left[\frac{z^3}{3}\right]_0^L - \frac{2}{L}\left[\frac{z^2}{2}\right]_0^L + \frac{1}{L^2}\left[\frac{z^3}{3}\right] + \right.$$

$$\left. + \left[z\right]_0^L - \frac{2}{L}\left[\frac{z^2}{2}\right]_0^L \right\} + \frac{\alpha \overline{\Delta T}}{h} \left\{ \frac{1}{L}\left[\frac{z^2}{2}\right]_0^L - \left[z\right]_0^L \right\} =$$

$$= \frac{1}{EI} \left\{ -\frac{3}{2}q\frac{L^3}{3} + \frac{3}{2}qL\frac{L^2}{2} - \frac{3}{2}q\frac{L^3}{3} - \frac{3}{2}qL^2L + \frac{3}{2}qL\frac{L^2}{2} + \frac{qL}{2}\frac{L^2}{2} - \frac{q}{2}\frac{L^3}{3} \right\} +$$

$$+ \frac{X}{EI} \left\{ \frac{1}{L^2}\frac{L^3}{3} + 2\left(\frac{1}{L^2}\frac{L^3}{3} + L - \frac{2}{L}\frac{L^2}{2}\right) \right\} + \frac{\alpha \overline{\Delta T}}{h} \left\{ \frac{1}{L} \cdot \frac{L^2}{2} - L \right\} =$$

$$= + \frac{qL^3}{EI} \left\{ -\frac{1}{2} + \frac{3}{4} - \frac{1}{2} - \frac{3}{2} + \frac{3}{4} + \frac{1}{4} - \frac{1}{6} \right\} + \frac{X}{EI} \left\{ + \frac{L}{3} + \frac{2}{3}L \right\} - \frac{\alpha \overline{\Delta T}}{h} \frac{L}{2} =$$

$$= \underbrace{-\frac{11}{12}\frac{qL^3}{EI} + \frac{XL}{EI} - \frac{\alpha \overline{\Delta T}}{h}\frac{L}{2}}$$

$\alpha_{ve} = \alpha_{vi}$ fornisce

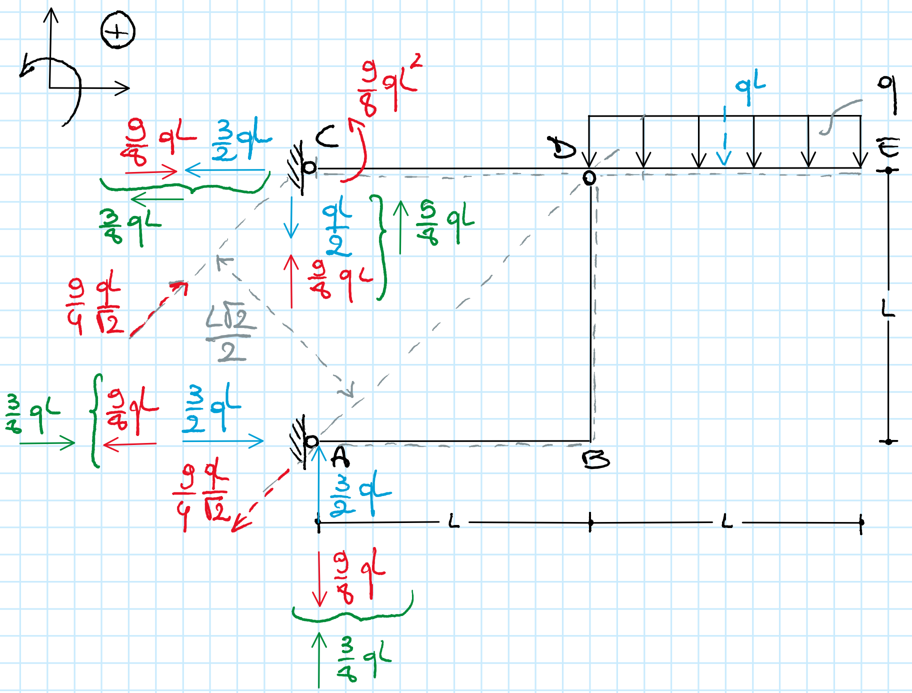
$$-\varepsilon X + \frac{M_1^0}{L} = -\frac{11}{12}\frac{qL^3}{EI} + \frac{XL}{EI} - \frac{\alpha \Delta T}{h}\frac{L}{2}$$

$$-\frac{L}{EI}X + \frac{qL^3}{3EI} = -\frac{11}{12}\frac{qL^3}{EI} + X\frac{L}{EI} - \frac{qL^3}{EI}$$

$$qL^3\left\{\frac{1}{3}+\frac{11}{12}+1\right\} = 2X \qquad X = \frac{9}{8}qL^2$$ POSITIVO

Verso ipotizzato ok!

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



TRATTO AB $0 \leq z \leq L$

$$\boxed{M^{(I)}(z) = \frac{3}{8} qLz}$$

$$\begin{cases} M_A = \emptyset \\ M_B = \frac{3}{8} qL^2 \end{cases}$$

TRATTO BD $0 \leq z \leq L$

$$\boxed{M^{(I)}(z) = \frac{3}{8} qL(L-z)}$$

$$\begin{cases} M_B = \frac{3}{8} qL^2 \\ M_D = \emptyset \end{cases}$$

TRATTO CD $0 \leq z \leq L$

$$\boxed{M^{(I)}(z) = \frac{5}{8} qLz - \frac{9}{8} qL^2}$$

$$\begin{cases} M_C = -\frac{9}{8} qL^2 \\ M_D = -\frac{qL^2}{2} \end{cases}$$

TRATTO DE $L \leq z \leq 2L$

$$\boxed{M^{(I)}(z) = -\frac{q}{2}(2L-z)^2}$$

$$\begin{cases} M_D = -\frac{qL^2}{2} \\ M_E = \emptyset \end{cases}$$

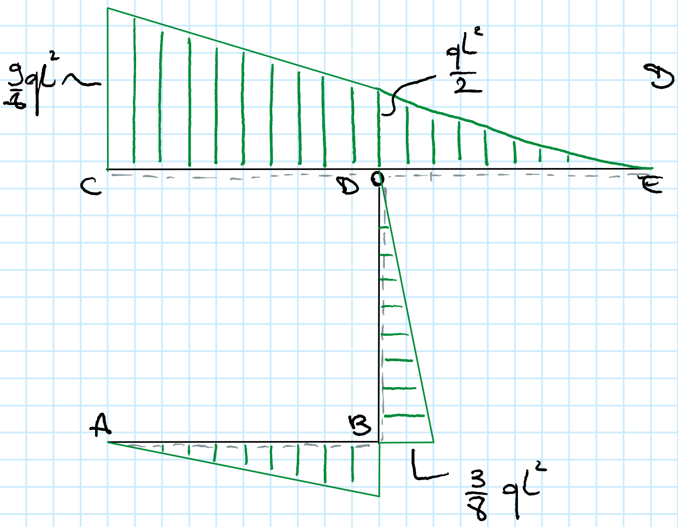
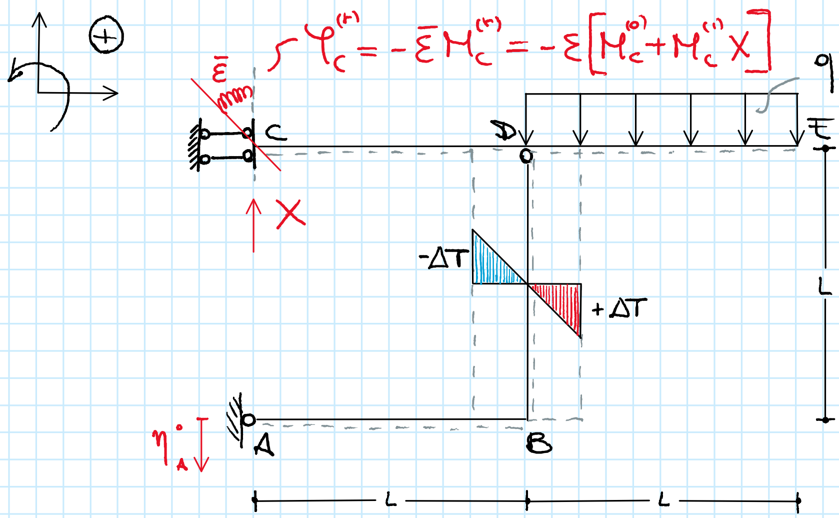


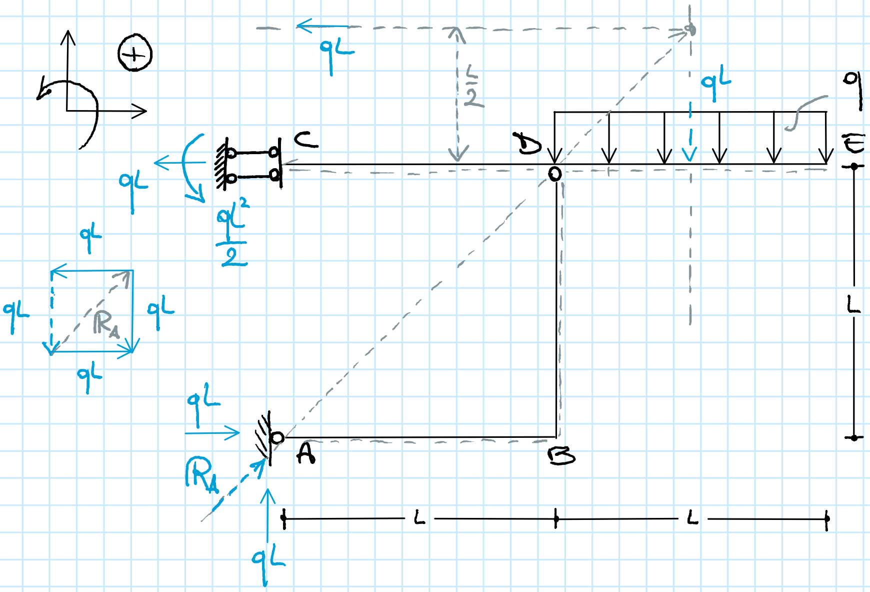
DIAGRAMMA $M^{(r)}(z)$

SOLUZIONE #2

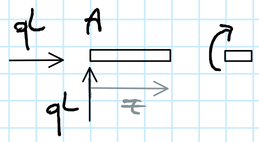
- SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



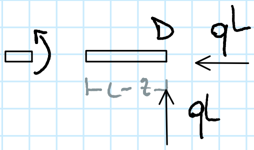
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qLz}$$

$$\begin{cases} M_A = \emptyset \\ M_B = qL^2 \end{cases}$$

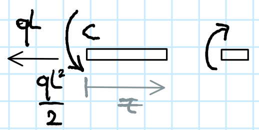
TRATTO BD $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL(L-z)}$$

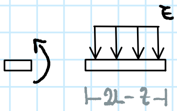
$$\begin{cases} M_B = qL^2 \\ M_D = \emptyset \end{cases}$$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2}}$$

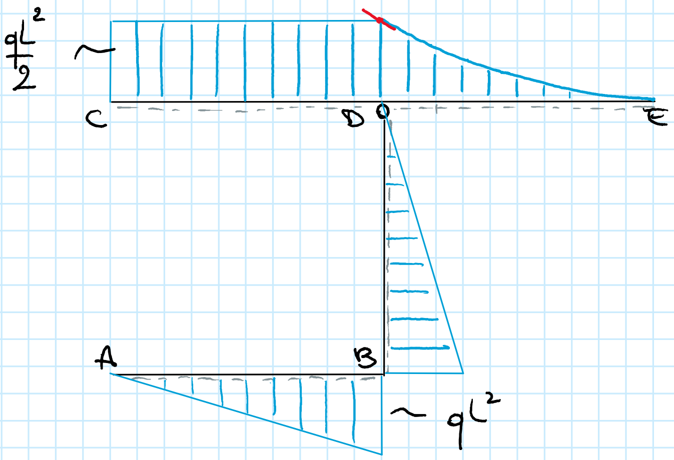
TRATTO DE $L \le z \le 2L$



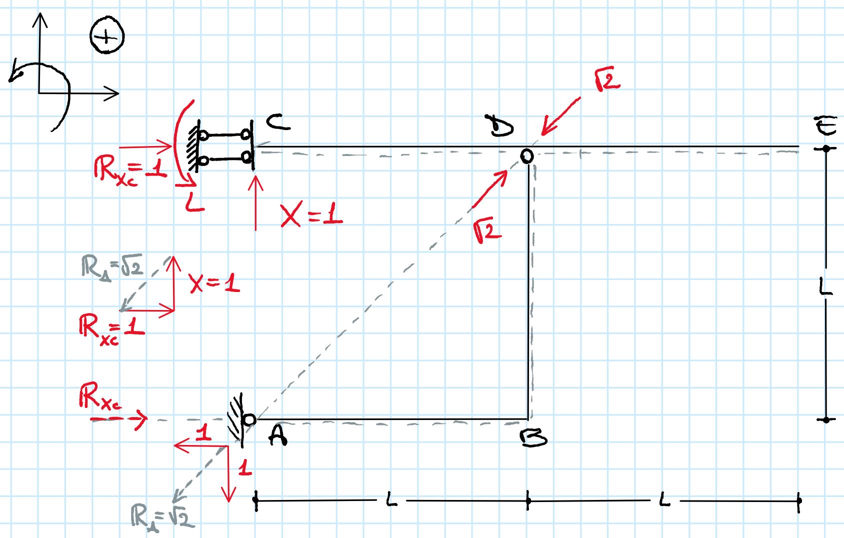
$$\boxed{M^{(0)}(z) = -\frac{q}{2}(2L-z)^2}$$

$$\begin{cases} M_D = -\frac{qL^2}{2} \\ M_E = \emptyset \end{cases}$$

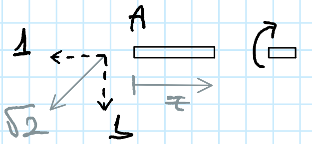
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO X = 1

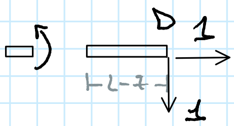


TRATTO AB $0 \le z \le L$



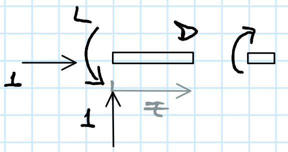
$$\boxed{M^{(1)}(z) = -z} \qquad \begin{cases} M_A = \emptyset \\ M_B = -L \end{cases}$$

TRATTO BD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -(L-z)} \qquad \begin{cases} M_B = -L \\ M_D = \emptyset \end{cases}$$

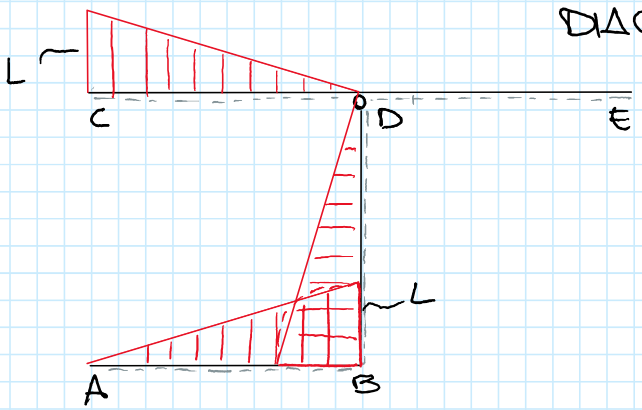
TRATTO CD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -L + z} \qquad \begin{cases} M_c = -L \\ M_D = \emptyset \end{cases}$$

TRATTO DE Scarico

DIAGRAMMA $M^{(1)}(z)$



$$-\bar{\varepsilon}\left[\overset{\frac{qL^2}{2}}{M_c^{(0)}} + X \overset{L}{M_c^{(1)}}\right]$$

$$\underline{L_{ve}} = 1 \cdot \eta_i + \sum_j R_j^{(1)} \eta_j^{(r)} = \underbrace{R_{y1}^{(1)}}_{-1} \cdot \left[-\eta_A^{\circ}\right] + \underbrace{M_c^{(1)}}_{L} \cdot \varphi_c^{(r)} =$$

$$= \underbrace{\eta_A^{\circ} - \bar{\varepsilon} L\left[\frac{qL^2}{2} + XL\right]}$$

$$\mathcal{L}_{vi} = \int_{Str} M^{(1)} \frac{M^{(h)}}{EI} \, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} \, dStr =$$

$$= \frac{1}{EI} \int_{Str} M^{(1)} M^{(0)} \, dSt + \frac{X}{EI} \int_{Str} \left[M^{(1)} \right]^2 dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} \, dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} M^{(1)} M^{(0)} dz + \int_{BD} M^{(1)} M^{(0)} dz + \int_{CD} M^{(1)} M^{(0)} dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_{AB} \left[M^{(1)} \right]^2 dz + \int_{BD} \left[M^{(1)} \right]^2 dz + \int_{CD} \left[M^{(1)} \right]^2 dz \right\} + \int_{BD} M^{(1)} \frac{\alpha \Delta T}{h} dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L \left[-z \right] \left[qLz \right] dz + \int_0^L qL \left[L - z \right] \left[z - L \right] dz + \right.$$

$$\left. + \int_0^L - \frac{qL^2}{2} \left[z - L \right] dz \right\} + \frac{X}{EI} \left\{ \int_0^L z^2 dz + \int_0^L (z-L)^2 dz + \int_0^L (z-L)^2 dz \right\} +$$

$$+ \frac{\alpha \Delta T}{h} \int_0^L (z - L) \, dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L - qLz^2 dz + \int_0^L qL \left[-z^2 - L^2 + 2zL \right] dz + \int_0^L - \frac{qL^2}{2} \left[z - L \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L z^2 dz + 2 \int_0^L (z^2 + L^2 - 2zL) \, dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L (z - L) \, dz =$$

$$= \frac{1}{EI} \left\{ -qL \cdot \frac{L^3}{3} + qL \left[-\frac{L^3}{3} - L^3 + 2 \frac{L^3}{2} \right] - \frac{qL^2}{2} \left[\frac{L^2}{2} - L^2 \right] \right\} +$$

$$+ \frac{X}{EI} \left\{ \frac{L^3}{3} + 2 \left[\frac{L^3}{3} + L^3 - 2 \frac{L^3}{2} \right] \right\} + \frac{\alpha \Delta T}{h} \left[\frac{L^2}{2} - L^2 \right] =$$

$$= \frac{1}{EI} \left\{ -\frac{qL^4}{3} - \frac{qL^4}{3} - qL^4 + qL^4 + \frac{qL^4}{4} \right\} + \frac{X}{EI} \left\{ \frac{L^3}{3} + \frac{2}{3} L^3 \right\} - \frac{L^2}{2} \frac{\alpha \Delta T}{h} =$$

$$= \underbrace{- \frac{5}{12} \frac{qL^4}{EI} + \frac{X}{EI} L^3 - \frac{\alpha \Delta T}{h} \frac{L^2}{2}}$$

$\mathcal{L}_{ve} = \mathcal{L}_{vi}$ fornisce

$$\eta_A^\circ - \bar{\varepsilon}L\left[\frac{qL^2}{2} + XL\right] = -\frac{5}{12}\frac{qL^4}{EI} + \frac{XL^3}{EI} - \frac{\alpha\Delta T}{h}\cdot\frac{L^2}{2}$$

$$\frac{qL^4}{3EI} - \frac{L^2}{EI}\cdot\frac{qL^2}{2} - \frac{L^3}{EI}X = -\frac{5}{12}\frac{qL^4}{EI} + \frac{L^3}{EI}X - \frac{qL^4}{EI}$$

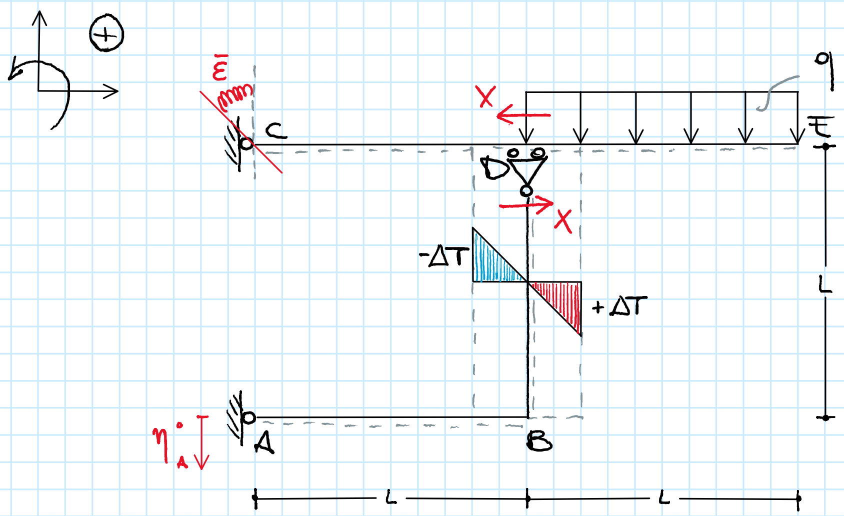
$$qL\left[\frac{1}{3} - \frac{1}{2} + \frac{5}{12} + 1\right] = X\,2$$

$$\frac{15}{12}qL = X\,2$$

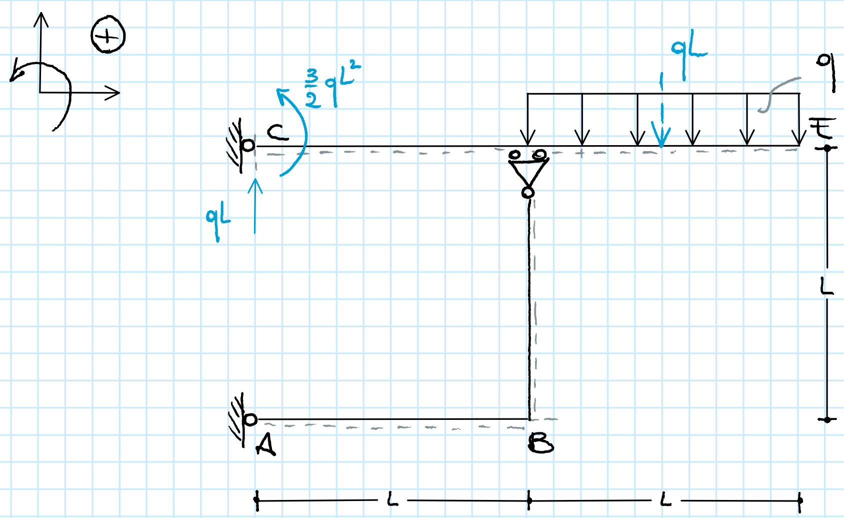
$X = \frac{5}{8}qL$ POSITIVO, verso corretto

SOLUZIONE #6

- SISTEMA PRINCIPALE ISOSTATICO

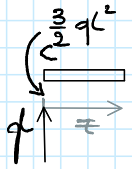


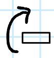
SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO AB
TRATTO BD } Scarichi $M^{(0)}(z) = \emptyset$

TRATTO CD $0 \le z \le L$

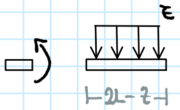




$$\boxed{M^{(0)}(z) = -\frac{3}{2}qL^2 + qLz}$$

$$\begin{cases} M_C = -\frac{3}{2}qL^2 \\ M_D = -\frac{qL^2}{2} \end{cases}$$

TRATTO DE $L \le z \le 2L$



$$\boxed{M^{(0)}(z) = -\frac{q}{2}(2L-z)^2}$$

$$\begin{cases} M_D = -\frac{qL^2}{2} \\ M_E = \emptyset \end{cases}$$

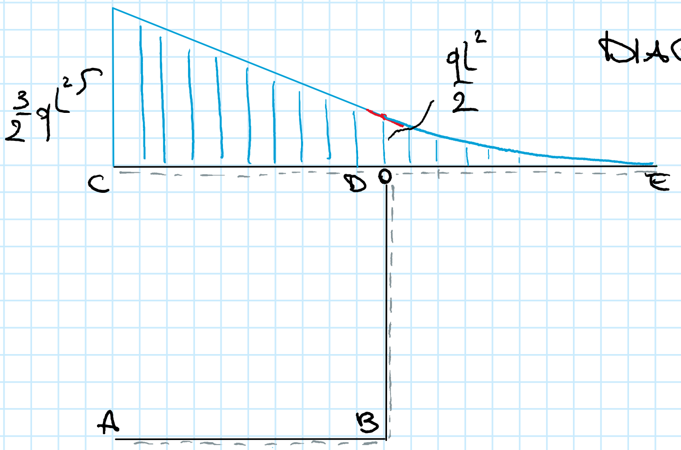
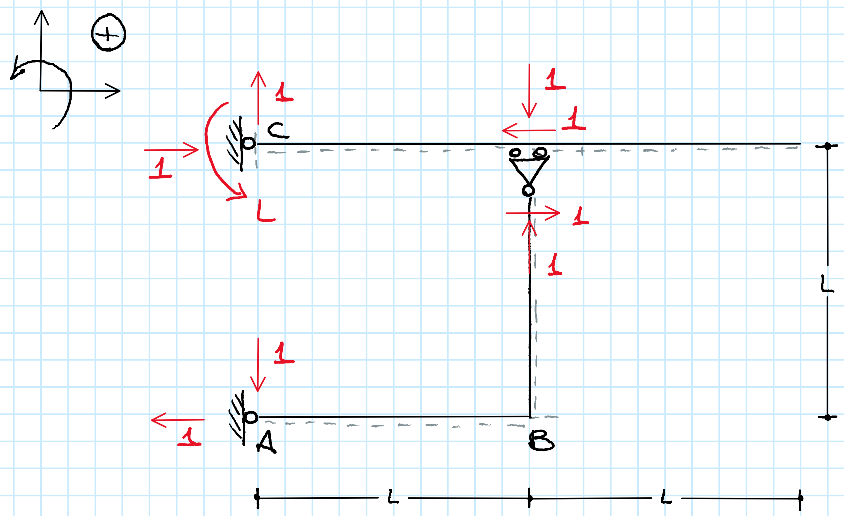
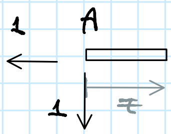


DIAGRAMMA $M^{(0)}(z)$

SCHEMA [1] SOLO $X = 1$



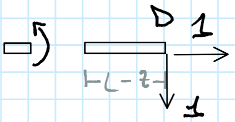
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -z}$$

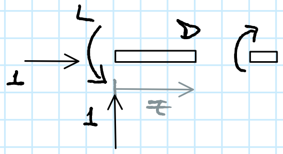
$$\begin{cases} M_A = \emptyset \\ M_B = -L \end{cases}$$

TRATTO BD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -(L-z)} \qquad \begin{cases} M_B = -L \\ M_D = \phi \end{cases}$$

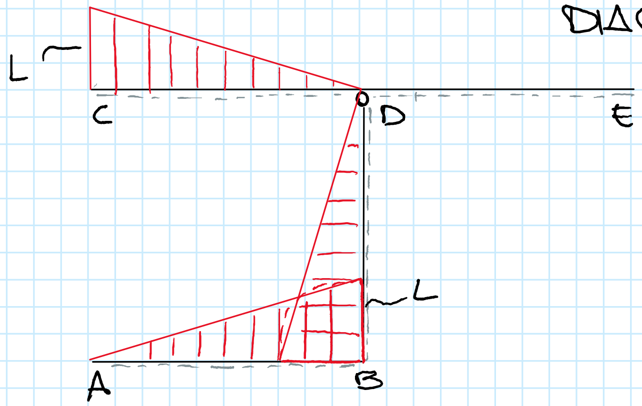
TRATTO CD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -L + z} \qquad \begin{cases} M_C = -L \\ M_D = \phi \end{cases}$$

TRATTO DE Scarico

DIAGRAMMA $M^{(1)}(z)$



$$\underbrace{\mathcal{L}_{ve}} = 1 \cdot M_D + \underbrace{\sum_j R_j^{(1)} \eta_j^{(r)}} = \underbrace{-\varepsilon\left[\frac{3}{2}qL^2 + XL\right] + M_A^0}$$

$$\underbrace{M_C^{(1)}}_{+L} \cdot \underbrace{\varphi_C^{(r)}}_{-\varepsilon M_C^{(r)}} + \underbrace{R_{yA}^{(1)}}_{-1} \underbrace{\eta_A^0}_{-M_A^0}$$

$$M_C^{(r)} = \underbrace{M_C^{(0)}}_{\frac{3}{2}qL^2} + X \underbrace{M_C^{(1)}}_{L}$$

$$\underbrace{\mathcal{L}_{vi}} = \int_{Str} M^{(1)} \frac{M^{(r)}}{EI}\, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$= \frac{1}{EI}\int_{Str} M^{(1)} M^{(0)}\, dStr + \frac{X}{EI}\int_{Str} \left[M^{(1)}\right]^2 dStr + \frac{\alpha\Delta T}{h}\int_{Str} M^{(1)}\, dStr$$

$$= \frac{1}{EI}\left\{\int_{CD} M^{(1)} M^{(0)}\, dz\right\} + \frac{X}{EI}\left\{\int_{AB}\left[M^{(1)}\right]^2 dz + \int_{BD}\left[M^{(1)}\right]^2 dz + \int_{CD}\left[M^{(1)}\right]^2 dz\right\} + \frac{\alpha\Delta T}{h}\int_{BD} M^{(1)}\, dz =$$

$$= \frac{1}{EI}\left\{\int_0^L \underbrace{(z-L)\left(qLz - \frac{3}{2}qL^2\right)}_{qLz^2 - \frac{3}{2}qL^2 z - qL^2 z + \frac{3}{2}qL^3} dz\right\} + \frac{X}{EI}\left\{\int_0^L z^2\, dz + \int_0^L \underbrace{(L-z)^2}_{L^2 + z^2 - 2Lz} dz + \int_0^L (z-L)^2\, dz\right\} +$$

$$+ \frac{\alpha\Delta T}{h}\int_0^L (z-L)\, dz =$$

$$= \frac{1}{EI}\left\{ qL\frac{L^3}{3} - \frac{3}{2}qL^2\cdot\frac{L^2}{2} - qL^2\frac{L^2}{2} + \frac{3}{2}qL^3\cdot L \right\} + \frac{X}{EI}\left\{ \frac{L^3}{3} + 2\left[L^3 + \frac{L^3}{3} - 2L\cdot\frac{L^2}{2} \right] \right\} + \frac{\alpha\Delta T}{h}\left\{ \frac{L^2}{2} - L^2 \right\} =$$

$$= \frac{qL^4}{EI}\left\{ \frac{1}{3} - \frac{3}{4} - \frac{1}{2} + \frac{3}{2} \right\} + \frac{X}{EI}L^3 - \frac{\alpha\Delta T}{h}\frac{L^2}{2} =$$

$$\underbrace{= \frac{7}{12}\frac{qL^4}{EI} + \frac{XL^3}{EI} - \frac{L^2}{2}\frac{\alpha\Delta T}{h}}$$

$\alpha_{ve} = \alpha_{vi}$ formica

$$-\frac{3}{2}qL^3\bar{\varepsilon} - \bar{\varepsilon}XL^2 + M_A^0 = \frac{7}{12}\frac{qL^4}{EI} + \frac{L^3}{EI}X - \frac{L^2}{2}\frac{\alpha\Delta T}{h}$$

$$-\frac{3}{2}\frac{qL^4}{EI} - \frac{L^3}{EI}X + \frac{qL^4}{3EI} = \frac{7}{12}\frac{qL^4}{EI} + \frac{L^3}{EI}X - \frac{qL^4}{EI}$$

$$qL\left\{ -\frac{3}{2} + \frac{1}{3} - \frac{7}{12} + 1 \right\} = 2X$$

$$-\frac{9}{12}qL = 2X$$

$X = -\frac{3}{8}qL$ NEGATIVO, verso opposto