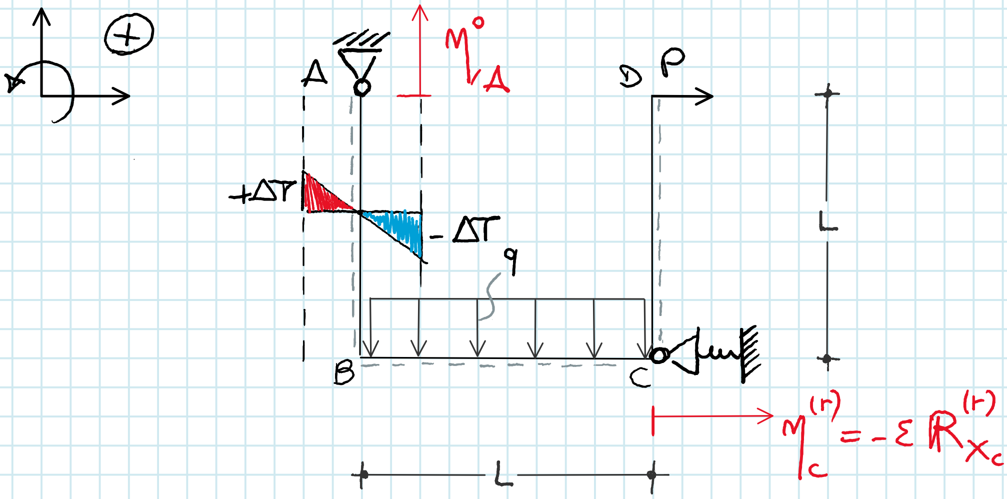
IPER 14

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA DETERMINANDO IL DIAGRAMMA DEI MOMENTI



POSIZIONI

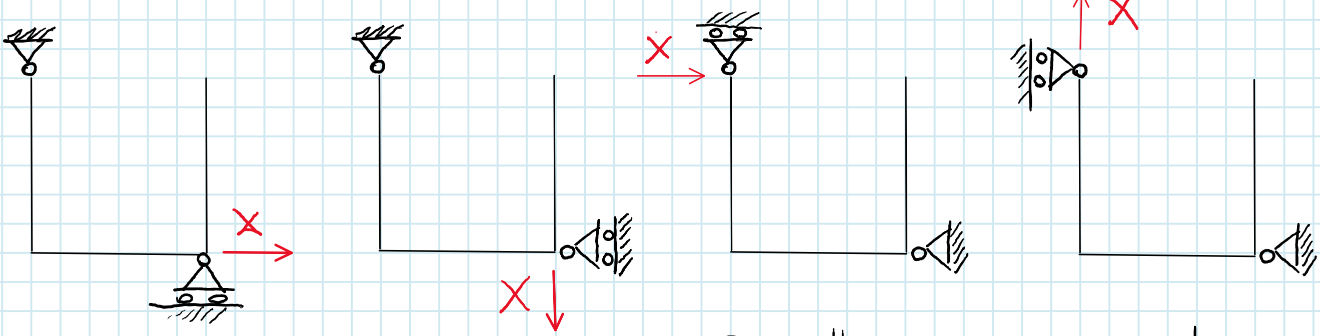
$$|P| = qL$$

$$|M_A^o| = \frac{19}{24}\frac{qL^4}{EI}$$

$$|\varepsilon| = \frac{1}{3}\frac{L^3}{EI}$$

$$\left|\frac{\alpha\Delta T}{h}\right| = \frac{qL^2}{EI}$$

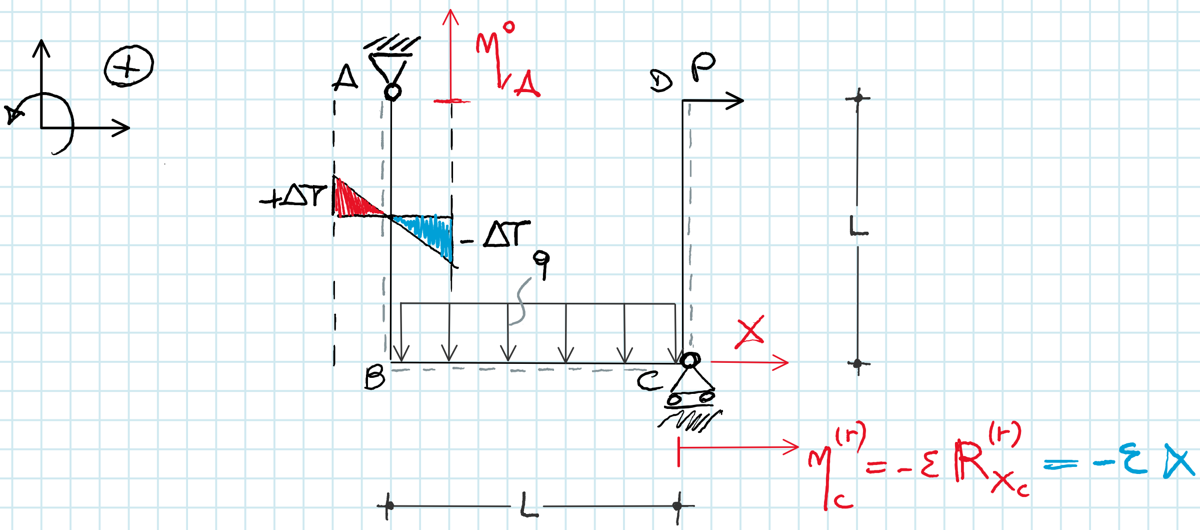
SOLUZIONI POSSIBILI:



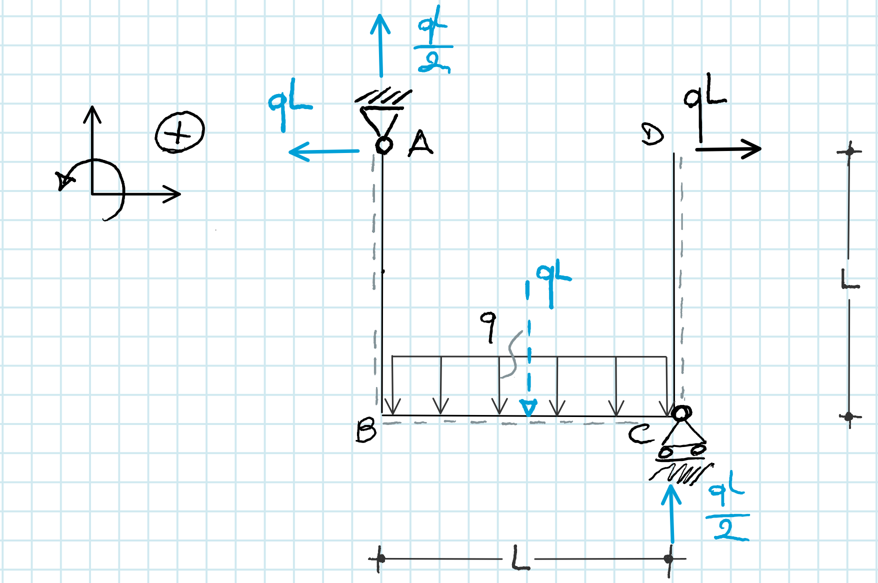
SOL. #1 SOL. #2 SOL. #3 SOL. #4

SOLUZIONE #1

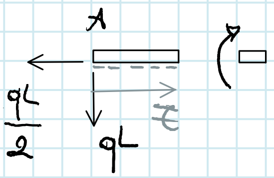
- SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



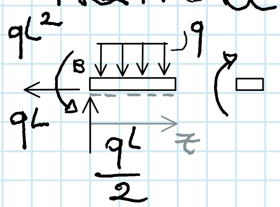
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -qL \cdot z}$$

$$\begin{cases} M_A = 0 \\ M_B = -qL^2 \end{cases}$$

TRATTO BC $0 \le z \le L$

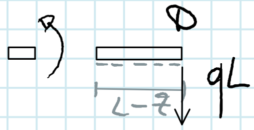


$$\boxed{M^{(0)}(z) = \frac{qL}{2} z - qL^2 - \frac{qz^2}{2}}$$

$$\begin{cases} M_B = -qL^2 \\ M_C = \frac{qL^2}{2} - qL^2 - \frac{qL^2}{2} = -qL^2 \end{cases}$$

↳ in mezzeria $= \frac{qL^2}{4} - qL^2 - \frac{qL^2}{8}$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -qL(L-z)}$$

$$\begin{cases} M_C = -qL^2 \\ M_D = \emptyset \end{cases}$$

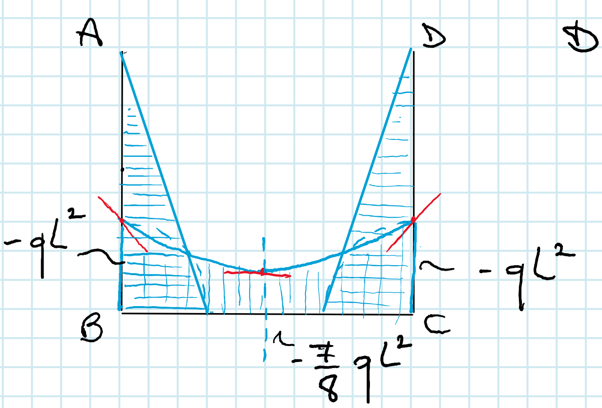
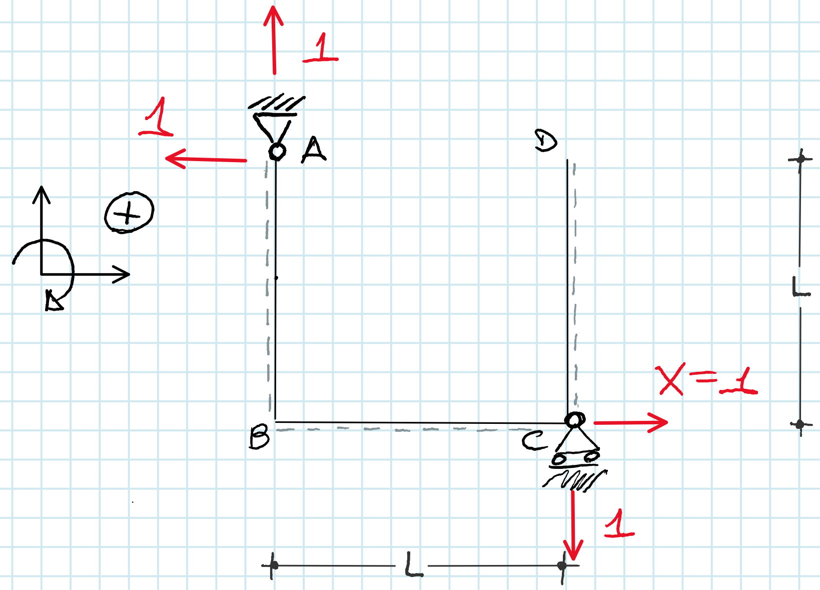
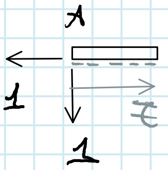


DIAGRAMMA $M^{(0)}(z)$

• SCHEMA [1] SOLO $X = 1$



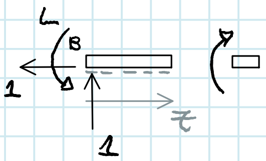
TRATTO AB $0 \leq z \leq L$



$$\boxed{M^{(1)}(z) = -z}$$

$$\begin{cases} M_A = 0 \\ M_B = -L \end{cases}$$

TRATTO BC $0 \leq z \leq L$



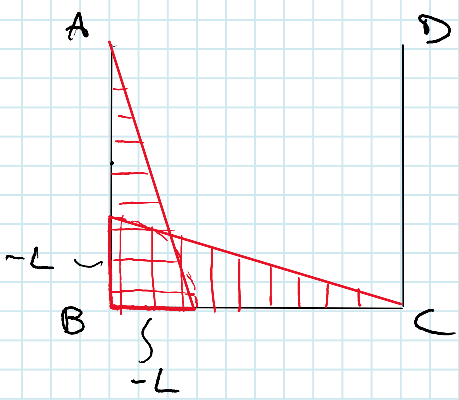
$$\boxed{M^{(1)}(z) = z - L}$$

$$\begin{cases} M_B = -L \\ M_C = \emptyset \end{cases}$$

TRATTO CD

$$\boxed{M^{(1)}(z) = \emptyset}$$

DIAGRAMMA $M^{(1)}(z)$



$$\underbrace{\mathcal{L}_{ve}} = X_i \cdot \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \{-\varepsilon X\} + R_{y_A}^{(1)} \cdot \eta_A^{\circ} =$$

$$= \underbrace{-\varepsilon X + \eta_A^{\circ}}$$

$$\underbrace{\mathcal{L}_{vi}} = \int_{Str} M^{(1)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr =$$

$$M^{(r)} = M^{(0)} + M^{(1)} X$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + X \int_{Str} \frac{[M^{(1)}]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} [-z][-qLz] dz + \int_{BC} [z-L] \left[\frac{qL}{2} z - qL^2 - \frac{qz^2}{2} \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_{AB} z^2 dz + \int_{BC} (z-L)^2 dz \right\} + \int_{AB} [-z] \left[+ \frac{\alpha \Delta T}{h} \right] dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L qLz^2 dz + \int_0^L \left[\frac{qL}{2} z^2 - qL^2 z - \frac{q}{2} z^3 - \frac{qL^2}{2} z + qL^3 + \frac{qL}{2} z^2 \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L z^2 dz + \int_0^L [z^2 + L^2 - 2zL] dz \right\} + \int_0^L -z \frac{\alpha \Delta T}{h} dz =$$

$$= \frac{1}{EI} \left\{ qL \left[\frac{z^3}{3} \right]_0^L + \frac{qL}{2} \left[\frac{z^3}{3} \right]_0^L - qL^2 \left[\frac{z^2}{2} \right]_0^L - \frac{q}{2} \left[\frac{z^4}{4} \right]_0^L - \frac{qL^2}{2} \left[\frac{z^2}{2} \right]_0^L + qL^3 [z]_0^L + \frac{qL}{2} \left[\frac{z^3}{3} \right]_0^L \right\}$$

$$+ \frac{X}{EI} \left\{ \left[\frac{z^3}{3} \right]_0^L + \left[\frac{z^3}{3} \right]_0^L + L^2 [z]_0^L - 2L \left[\frac{z^2}{2} \right]_0^L \right\} - \frac{\alpha \Delta T}{h} \left[\frac{z^2}{2} \right]_0^L =$$

$$= \frac{1}{EI}\left\{ \frac{qL^4}{3} + \frac{qL^4}{6} - \frac{qL^4}{2} - \frac{qL^4}{8} - \frac{qL^2}{4} + qL^4 + \frac{qL^4}{6} \right\} +$$

$$+ \frac{X}{EI}\left\{ \frac{L^3}{3} + \frac{L^3}{3} + \cancel{L^3} - \cancel{L^3} \right\} - \alpha \frac{\overline{\Delta T}}{h} \frac{L^2}{2}$$

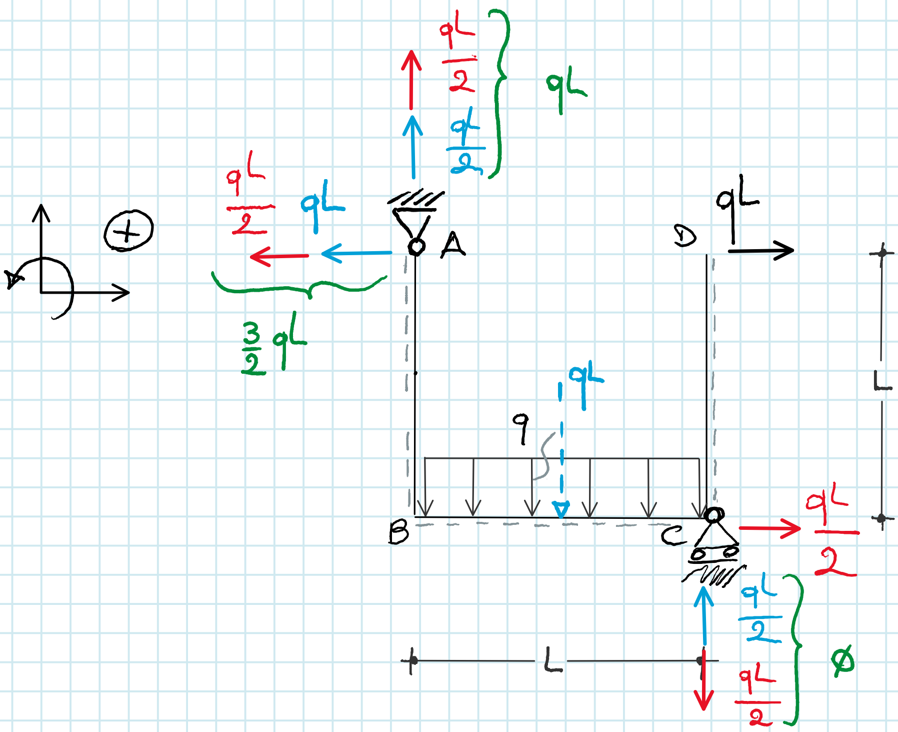
$$= \underbrace{\frac{1}{EI} \frac{19}{24} qL^4 + \frac{X}{EI} \frac{2}{3} L^3 - \frac{\alpha \Delta T}{h} \frac{L^2}{2}}$$

$\mathcal{L}_{ve} = \mathcal{L}_{vi}$ formisee

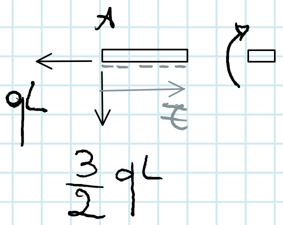
$$-\underbrace{\varepsilon}_{\frac{1}{3}\frac{L^3}{EI}} X + \cancel{\underbrace{\eta_1^0}_{\frac{19}{24}\frac{qL^4}{EI}}} = \cancel{\frac{1}{EI} \frac{19}{24} qL^4} + \frac{X}{EI} \frac{2}{3} L^3 - \alpha \underbrace{\frac{\overline{\Delta T}}{h}}_{\frac{qL^2}{EI}} \frac{L^2}{2}$$

$$X \frac{L^3}{EI} = \frac{q}{2} \frac{L^4}{EI} \longrightarrow X = \frac{qL}{2} \quad \text{POSITIVA}$$

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



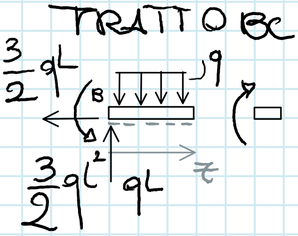
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(H)}(z) = -\frac{3}{2}qLz}$$

$$\begin{cases} M_A = \emptyset \\ M_B = -\frac{3}{2}qL \end{cases}$$

TRATTO BC $0 \le z \le L$



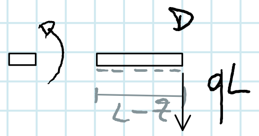
$$\boxed{M^{(H)}(z) = qLz - q\frac{z^2}{2} - \frac{3}{2}qL^2}$$

$$\begin{cases} M_B = -\frac{3}{2}qL^2 \\ M_C = qL^2 - \frac{qL^2}{2} - \frac{3}{2}qL^2 = -qL^2 \end{cases}$$

in mezzeria:

$$M^{(H)}\left(\frac{L}{2}\right) = \frac{qL^2}{2} - \frac{qL^2}{8} - \frac{3}{2}qL^2 = -\frac{9}{8}qL^2$$

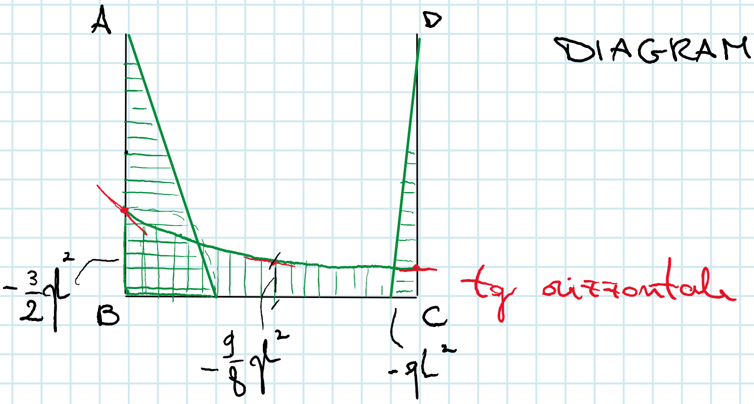
TRATTO CD $0 \le z \le L$



$$\boxed{M^{(H)}(z) = -qL(L-z)}$$

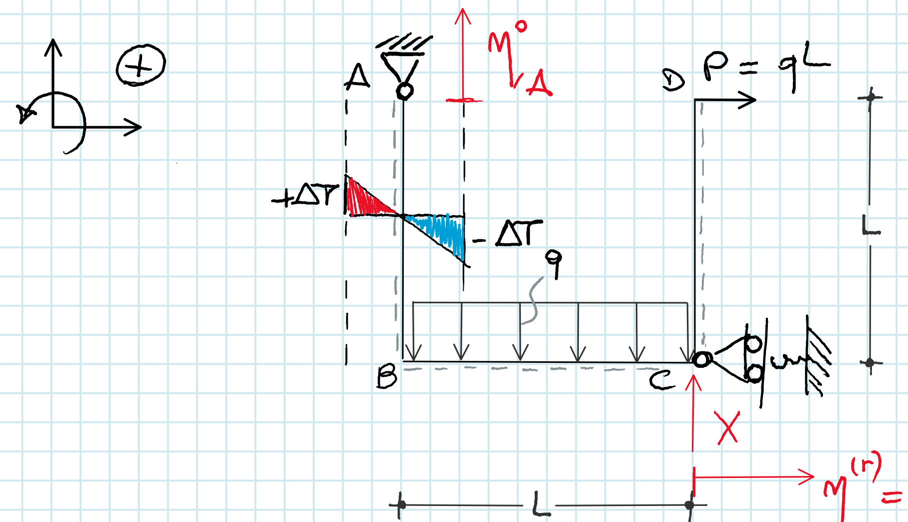
$$\begin{cases} M_C = -qL^2 \\ M_D = \emptyset \end{cases}$$

DIAGRAMMA FINALE $M^{(r)}(z)$



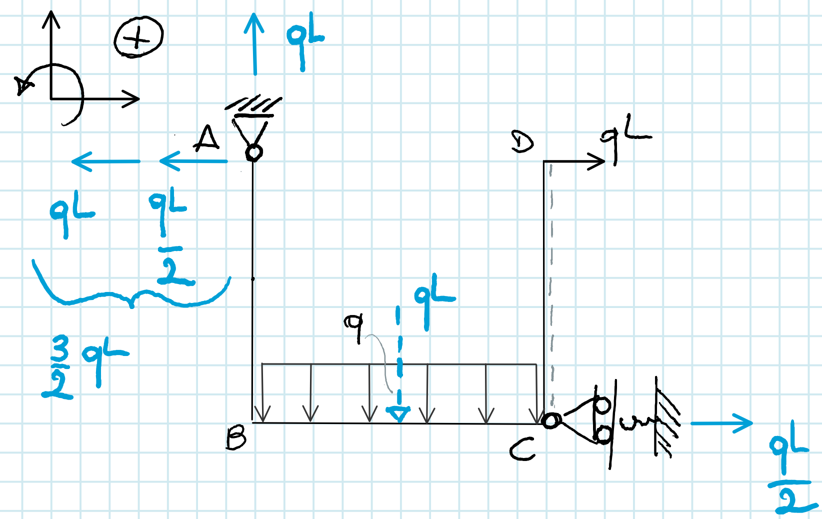
SOLUZIONE # 2

- **SISTEMA PRINCIPALE ISOSTATICO**

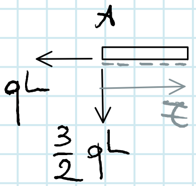


$$M_c^{(r)} = -\varepsilon R_{X_c}^{(r)} = R_{X_c}^{(0)} + R_{X_c}^{(1)} X$$

SCHEMA [0] SOLO CARICHI ESTERNI



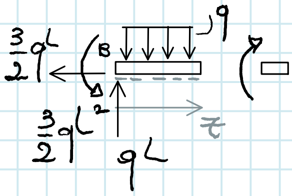
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{3}{2} qL z}$$

$$\begin{cases} M_A = 0 \\ M_B = -\frac{3}{2} qL^2 \end{cases}$$

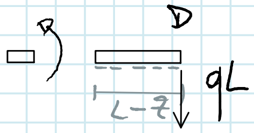
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL\cdot z - q\frac{z^2}{2} - \frac{3}{2} qL^2}$$

$$\begin{cases} M_B = -\frac{3}{2} qL^2 \\ M_C = qL^2 - \frac{qL^2}{2} - \frac{3}{2} qL^2 = -qL^2 \end{cases}$$

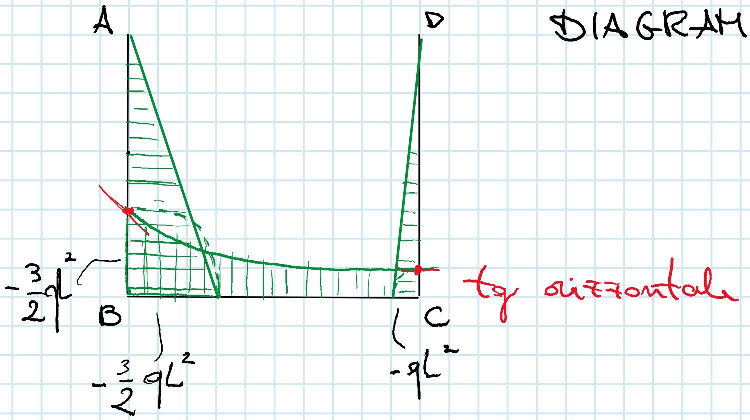
TRATTO CD $0 \le z \le L$



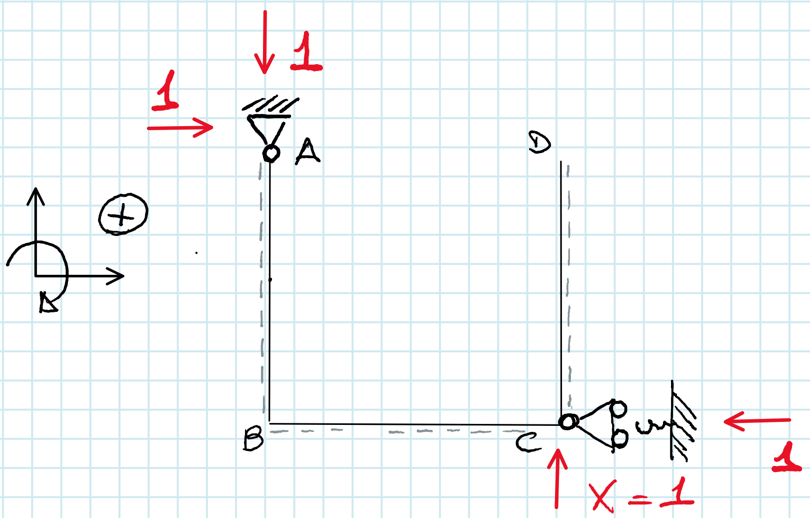
$$\boxed{M^{(0)}(z) = -qL(L-z)}$$

$$\begin{cases} M_C = -qL^2 \\ M_D = \emptyset \end{cases}$$

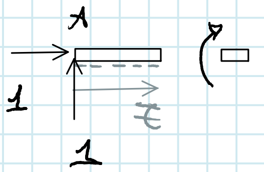
DIAGRAMMA $M^{(0)}(z)$



• SCHEMA [1] SOLO $X = 1$



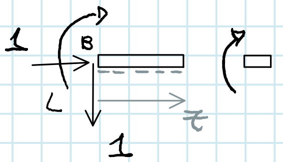
TRATTO AB $\quad 0 \leq z \leq L$



$$\boxed{M^{(1)}(z) = z}$$

$$\begin{cases} M_A = \emptyset \\ M_B = L \end{cases}$$

TRATTO BC $\quad 0 \leq z \leq L$

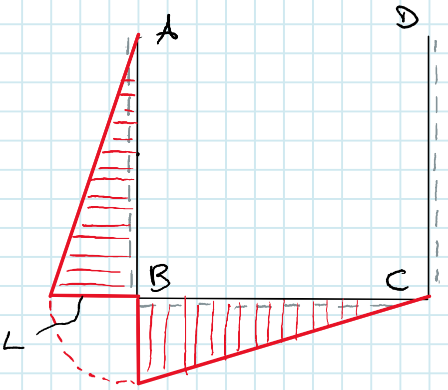


$$\boxed{M^{(1)}(z) = L - z}$$

$$\begin{cases} M_B = L \\ M_C = \emptyset \end{cases}$$

TRATTO CD

$$\boxed{M^{(1)}(z) = \emptyset}$$



$$-\varepsilon\Big[\underbrace{R^{(0)}_{x_c}}_{\frac{qL}{2}} + \underbrace{R^{(1)}_{x_c}}_{-1} \cdot X\Big]$$

$$\underline{L_{ve}} = X_i \cdot \eta_i^{(r)} + \sum_j R_j^{(1)} \, \eta_j^{(r)} =$$

$$= 1 \cdot \emptyset + \underbrace{R^{(1)}_{y_A}}_{-1} \cdot \eta^0_A + \underbrace{R^{(1)}_{x_c}}_{-1} \, \eta^{(r)}_c = \underbrace{-\eta^0_A + \varepsilon\left[\frac{qL}{2} - X\right]}$$

$$\mathcal{L}_{vi} = \int_{Str} M^{(1)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$= \frac{1}{EI} \int_{Str} M^{(1)} M^{(0)} dStr + \frac{X}{EI} \int_{Str} \left[M^{(1)}\right]^2 dStr + \frac{\alpha \overline{\Delta T}}{h} \int_{Str} M^{(1)} dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} M^{(1)} M^{(0)} dStr + \int_{BC} M^{(1)} M^{(0)} dStr \right\} + \frac{X}{EI} \left\{ \int_{AB} \left[M^{(1)}\right]^2 dz + \int_{BC} \left[M^{(1)}\right]^2 dz \right\} +$$

$$+ \frac{\alpha \overline{\Delta T}}{h} \int_{AB} M^{(1)} dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L z \left[-\frac{3}{2} qLz \right] dz + \int_0^L [L-z] \left[qLz - \frac{3}{2} qL^2 - \frac{q}{2} z^2 \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L z^2 dz + \int_0^L \underbrace{[L-z]^2}_{L^2+z^2-2Lz} dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_0^L z\, dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L -\frac{3}{2} qLz^2 dz + \int_0^L \left[qL^2 z - \frac{3}{2} qL^3 - \frac{qL}{2} z^2 - qLz^2 + \frac{3}{2} qL^2 z + \frac{q}{2} z^3 \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L z^2 dz + \int_0^L \left[L^2 + z^2 - 2Lz \right] dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_0^L z\, dz =$$

$$= \frac{1}{EI} \left\{ -\frac{3}{2} qL \left[\frac{z^3}{3} \right]_0^L + qL^2 \left[\frac{z^2}{2} \right]_0^L - \frac{3}{2} qL^3 [z]_0^L - \frac{qL}{2} \left[\frac{z^3}{3} \right]_0^L - qL \left[\frac{z^3}{3} \right]_0^L + \right.$$

$$\left. \frac{3}{2} qL^2 \left[\frac{z^2}{2} \right]_0^L + \frac{q}{2} \left[\frac{z^4}{4} \right]_0^L \right\} + \frac{X}{EI} \left\{ \left[\frac{z^3}{3} \right]_0^L + L^2 [z]_0^L + \left[\frac{z^3}{3} \right]_0^L - 2L \left[\frac{z^2}{2} \right]_0^L \right\} +$$

$$+ \frac{\alpha \overline{\Delta T}}{h} \left[\frac{z^2}{2} \right]_0^L =$$

$$= \frac{1}{EI} \left\{ -\frac{3}{6} qL^4 + \frac{1}{2} qL^4 - \frac{3}{2} qL^4 - \frac{qL^4}{6} - \frac{1}{3} qL^4 + \frac{3}{4} qL^4 + \frac{1}{8} qL^4 \right\} +$$

$$+ \frac{X}{EI} \left\{ \frac{1}{3} L^3 + \cancel{L^3} + \frac{1}{3} L^3 - \cancel{L^3} \right\} + \frac{\alpha \overline{\Delta T}}{h} \frac{L^2}{2} =$$

$$= \frac{qL^4}{EI}\left\{\frac{-12+12-36-4-8+18+3}{24}\right\} + \frac{2}{3}\frac{L^3}{EI}X + \frac{\alpha\Delta T}{h}\frac{L^2}{2}$$

$$= \underbrace{-\frac{9}{8}\frac{qL^4}{EI} + \frac{2}{3}\frac{L^3}{EI}X + \frac{\alpha\Delta T}{h}\frac{L^2}{2}}$$

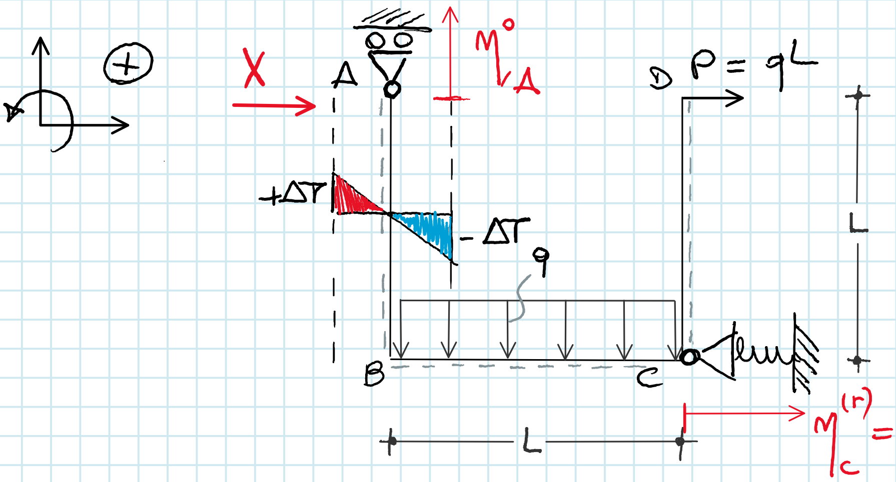
$L_{ve} = L_{vi}$ fornisce:

$$\underbrace{-\underbrace{\eta_A^0}_{-\frac{19}{24}\frac{qL^4}{EI}} + \frac{qL^4}{6EI}}_{-\frac{15}{24}\frac{qL^4}{EI}} + \varepsilon\frac{qL}{2} - \varepsilon X = \overbrace{-\frac{9}{8}}^{-\frac{27}{24}}\frac{qL^4}{EI} + \frac{2}{3}\frac{L^3}{EI}X + \underbrace{\frac{\alpha\Delta T}{h}\frac{L^2}{2}}_{\frac{12}{24}\frac{qL^4}{EI}}$$

$$X\left[\frac{1}{3}\frac{L^3}{EI} + \frac{2}{3}\frac{L^3}{EI}\right] = \emptyset \qquad X = \emptyset \quad \text{ok!}$$

SOLUZIONE # 3

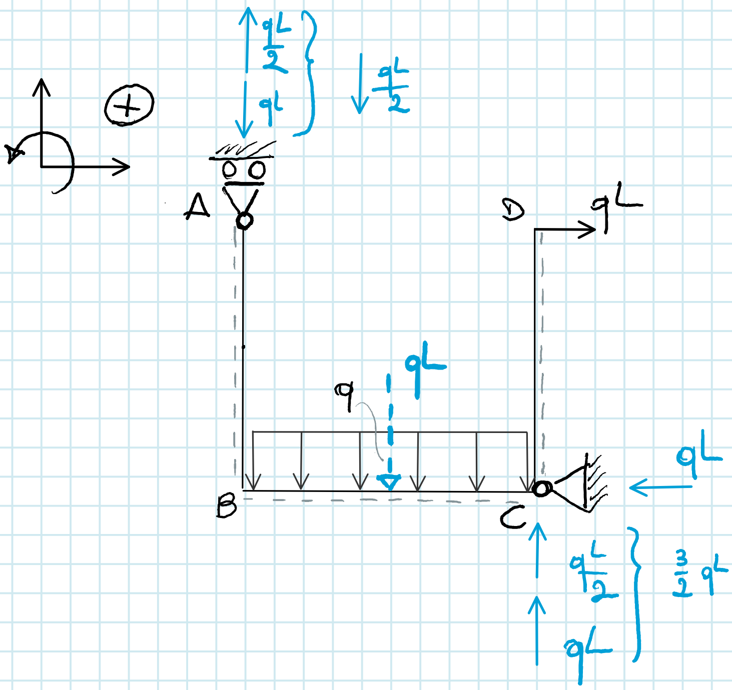
- SISTEMA PRINCIPALE ISOSTATICO



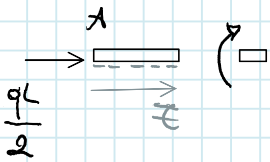
$M_c^{(r)} = -\varepsilon\, R_{X_c}^{(r)}$

$R_{X_c}^{(0)} + R_{X_c}^{(1)} X$

SCHEMA [0] SOLO CARICHI ESTERNI

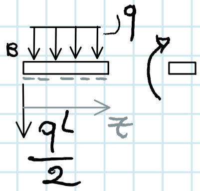


TRATTO AB $0 \le z \le L$



$\boxed{M^{(0)}(z) = \emptyset}$

TRATTO BC $0 \le z \le L$

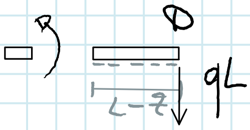


$$\boxed{M^{(0)}(z) = -\frac{qL}{2}z - \frac{qz^2}{2}}$$

$$\begin{cases} M_B = \varnothing \\ M_C = -qL^2 \end{cases}$$

↳ in mezzeria $= -\frac{qL^2}{4} - \frac{qL^2}{2} = \frac{5}{4}qL^2$

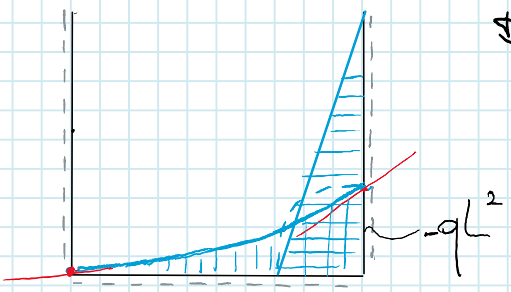
TRATTO CD $0 \le z \le L$



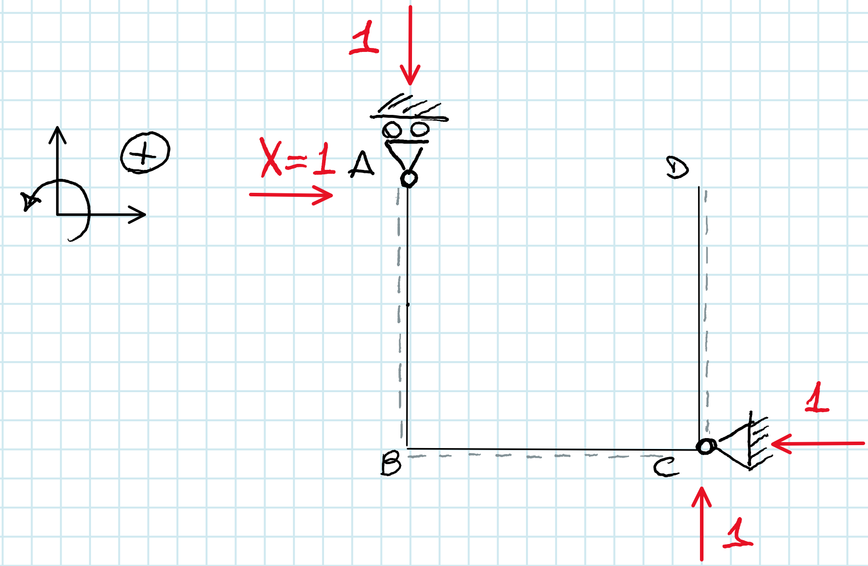
$$\boxed{M^{(0)}(z) = -qL(L-z)}$$

$$\begin{cases} M_C = -qL^2 \\ M_D = \varnothing \end{cases}$$

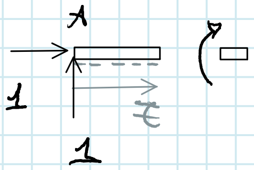
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $X = 1$



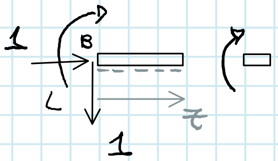
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(1)}(z) = z}$$

$$\begin{cases} M_A = \varnothing \\ M_B = L \end{cases}$$

TRATTO BC $0 \le z \le L$

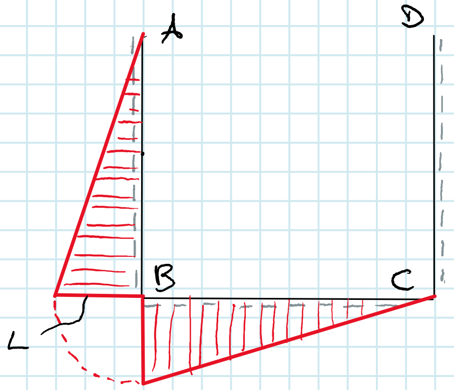


$$\boxed{M^{(1)}(z) = L - z}$$

$$\begin{cases} M_B = L \\ M_C = \varnothing \end{cases}$$

TRATTO CD

$$\boxed{M^{(1)}(z) = \varnothing}$$



$$\underbrace{\mathcal{L}_{ve}} = X_i \cdot \eta_i^{(r)} + \sum_j \mathbb{R}_j^{(1)} \eta_j^{(r)} = \quad -\varepsilon\Big[\underbrace{\mathbb{R}_{x_c}^{(0)}}_{-qL} + \underbrace{\mathbb{R}_{x_c}^{(1)}}_{-1}\cdot X\Big]$$

$$= 1\cdot\phi + \underbrace{\mathbb{R}_{y_A}^{(1)}}_{-1}\cdot \eta_A^{0} + \underbrace{\mathbb{R}_{x_c}^{(1)}}_{-1}\cdot \eta_c^{(r)} = \underbrace{-\eta_A^{0} - \varepsilon\left[qL + X\right]}$$

$$\underbrace{\mathcal{L}_{vi}} = \int_{Str} M^{(1)} \frac{M^{(r)}}{EI}\, dStr + \int_{Str} M^{(1)} \frac{\alpha\overline{\Delta T}}{h}\, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI}\, dStr + X\int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha\overline{\Delta T}}{h}\, dStr =$$

$$= \frac{1}{EI}\int_{\overline{BC}} M^{(1)} M^{(0)} + \frac{X}{EI}\left\{\int_{\overline{AB}} \left[M^{(1)}\right]^2 dz + \int_{\overline{BC}} \left[M^{(1)}\right]^2 dz\right\} + \int_{\overline{AB}} M^{(1)} \frac{\alpha\overline{\Delta T}}{h}\, dz =$$

$$= \frac{1}{EI}\left\{\int_0^L (L-z)\left[\frac{qL}{2}z - \frac{qz^2}{2}\right] dz\right\} + \frac{X}{EI}\left\{\int_0^L z^2 dz + \int_0^L \underbrace{\left[L-z\right]^2}_{L^2+z^2-2Lz} dz\right. +$$

$$\left. + \frac{\alpha\overline{\Delta T}}{h}\int_0^L z\, dz = \right.$$

$$= \frac{1}{EI}\left\{\int_0^L \left[-\frac{qL^2}{2}z - \frac{qL}{2}z^2 + \frac{qL}{2}z^2 + \frac{q}{2}z^3\right] dz\right\} +$$

$$+ \frac{X}{EI}\left\{\left[\frac{z^3}{3}\right]_0^L + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right\} + \frac{\alpha\overline{\Delta T}}{h}\left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{1}{EI}\left\{-\frac{qL^2}{2}\left[\frac{z^2}{2}\right]_0^L + \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L\right\} + \frac{X}{EI}\left\{\frac{L^3}{3} + L^3 + \frac{L^3}{3} - L^3\right\} + \frac{\alpha\overline{\Delta T}}{h}\frac{L^2}{2} =$$

$$= \underbrace{\frac{1}{EI}\left\{-\frac{qL^4}{4} + \frac{qL^4}{8}\right\} + \frac{X}{EI}\cdot\frac{2}{3}L^3 + \frac{\alpha\overline{\Delta T}}{h}\frac{L^2}{2}}$$

$\mathcal{L}_{ve} = \mathcal{L}_{vi}$ fornisce

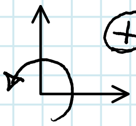
$$\underbrace{-\eta_1^{\circ}}_{\frac{19}{24}\frac{qL^4}{EI}} - \underbrace{\varepsilon}_{\frac{L^3}{3EI}}[qL] - \varepsilon X = -\frac{qL^4}{8EI} + \frac{2L^3}{3EI}X + \underbrace{\frac{\alpha \Delta T}{h}}_{\frac{qL^2}{EI}}\frac{L^2}{2}$$

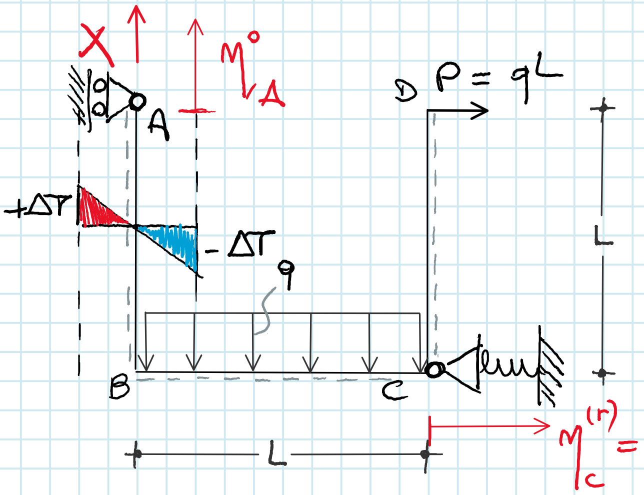
$$-\frac{19}{24}\frac{qL^4}{EI} - \frac{qL^4}{3EI} + \frac{qL^4}{8EI} - \frac{qL^4}{2EI} = X\left[\frac{2L^3}{3EI} + \frac{L^3}{3EI}\right]$$

$X = -\frac{3}{2}qL$ negativa, verso sx!

SOLUZIONE #4

- SISTEMA PRINCIPALE ISOSTATICO

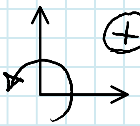


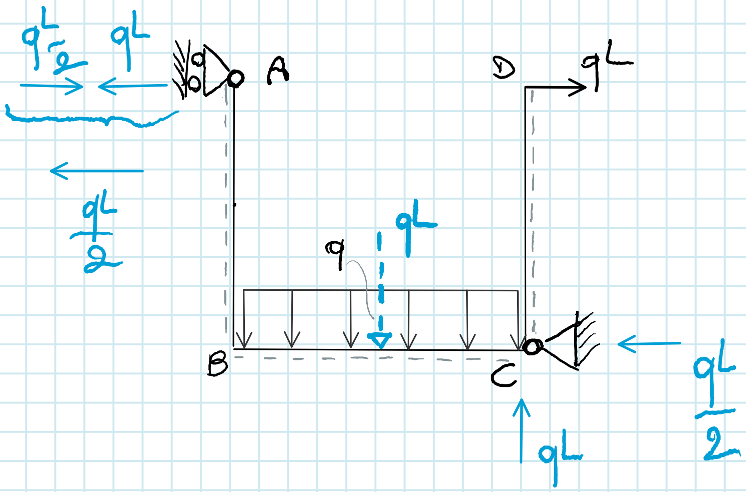


$M_c^{(r)} = -\varepsilon R_{X_c}^{(r)}$

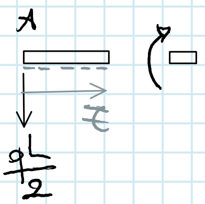
$R_{X_c}^{(0)} + R_{X_c}^{(1)} X$

SCHEMA [0] SOLO CARICHI ESTERNI





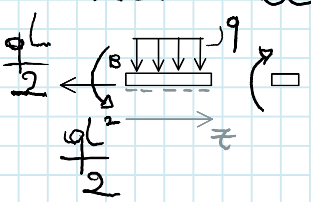
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{qL}{2} z}$$

$$\begin{cases} M_A = 0 \\ M_B = -\dfrac{qL^2}{2} \end{cases}$$

TRATTO BC $0 \le z \le L$

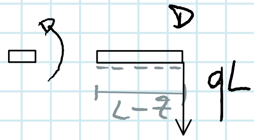


$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2} - \frac{qz^2}{2}}$$

$$\begin{cases} M_B = -\dfrac{qL^2}{2} \\ M_C = -qL^2 \end{cases}$$

↳ in mezzeria $M^{(0)}\left(\frac{L}{2}\right) = -\frac{qL^2}{2} - \frac{qL^2}{8} = -\frac{5}{8} qL^2$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -qL(L-z)}$$

$$\begin{cases} M_C = -qL^2 \\ M_D = \varnothing \end{cases}$$

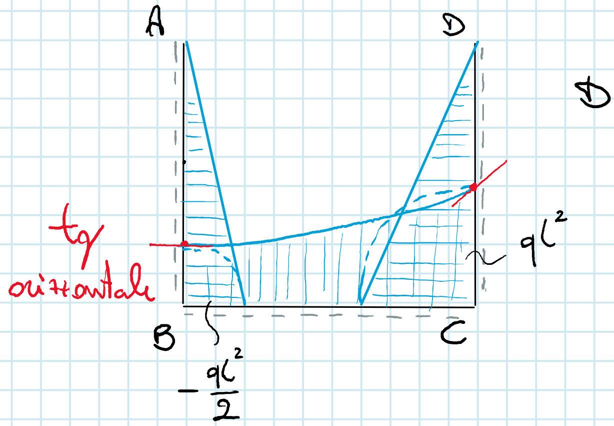
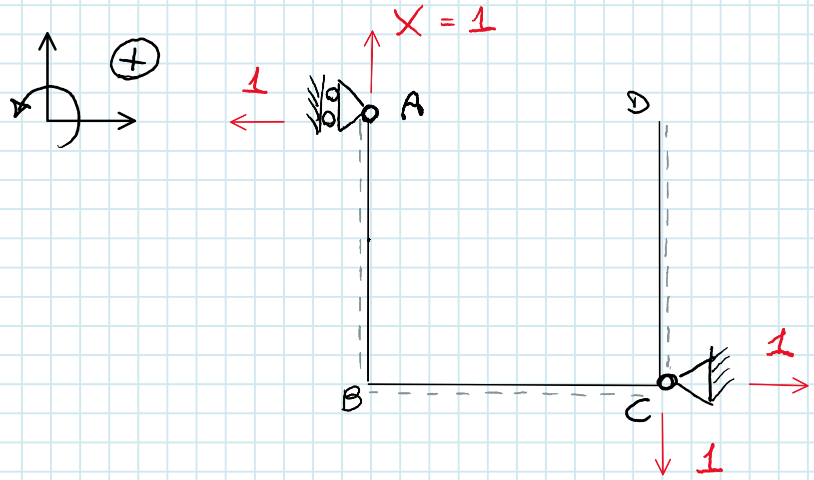
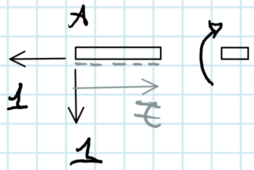


DIAGRAMMA $M^{(0)}(z)$

• SCHEMA [1] SOLO X = 1



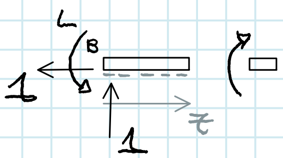
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -z}$$

$$\begin{cases} M_A = 0 \\ M_B = -L \end{cases}$$

TRATTO BC $0 \le z \le L$



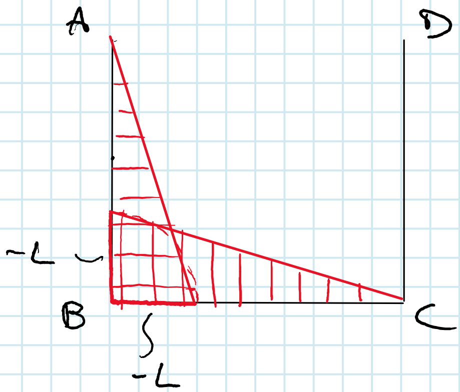
$$\boxed{M^{(1)}(z) = z - L}$$

$$\begin{cases} M_B = -L \\ M_C = \varnothing \end{cases}$$

TRATTO CD

$$\boxed{M^{(1)}(z) = \emptyset}$$

DIAGRAMMA $M^{(1)}(z)$



$$\underline{\mathcal{L}_{ve}} = X_i \cdot \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$= 1 \cdot \eta_A^{\circ} + \underbrace{R_{x_c}^{(1)}}_{1} \cdot \underbrace{\eta_c^{(r)}}_{-\varepsilon \underbrace{R_{x_c}^{(r)}}_{R_{x_c}^{(0)} + R_{x_c}^{(1)} X}} = \eta_A^{\circ} - \varepsilon \underbrace{\left[-\frac{qL}{2} + X \right]}$$

$$\underline{\mathcal{L}_{vi}} = \int_{Str} M^{(1)} \frac{M^{(r)}}{EI} \, dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} \, dStr + \frac{X}{EI} \int_{Str} \left[M^{(1)} \right]^2 dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} \left[-z \right] \left[-\frac{qL}{2} z \right] dz + \int_{BC} (z - L) \left[-\frac{qL^2}{2} - \frac{qz^2}{2} \right] \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_{AB} z^2 \, dz + \int_{BC} (z - L)^2 \, dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_{AB} \left[-z \right] dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L \frac{qL}{2} z^2 \, dz + \int_0^L \left[-\frac{qL^2}{2} z - \frac{qz^3}{2} + \frac{qL^3}{2} + \frac{qL}{2} z^2 \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L z^2 \, dz + \int_0^L \left[z^2 + L^2 - 2Lz \right] dz \right\} - \frac{\alpha \overline{\Delta T}}{h} \int_0^L z \, dz =$$

$$= \frac{1}{EI}\left\{ \frac{qL^4}{6} - \frac{qL^4}{4} - \frac{qL^4}{8} + \frac{qL^4}{2} + \frac{qL^4}{6} \right\} + \frac{X}{EI}\left\{ \frac{L^3}{3} + \frac{L^3}{3} + L^3 - \frac{2L^3}{2} \right\} - \frac{\alpha\Delta T}{h}\frac{L^2}{2} =$$

$$= \underbrace{\frac{qL^4}{EI}\frac{11}{24} + \frac{2L^3}{3EI}X - \frac{\alpha\Delta T}{h}\frac{L^2}{2}}$$

$\mathcal{L}_{ve} = \mathcal{L}_{vi}$ fornisce:

$$\frac{qL^2}{EI}$$

$$\eta_A^0 + \varepsilon\frac{qL}{2} - \varepsilon X = \frac{qL^4}{EI}\frac{11}{24} + \frac{2L^3}{3EI}X - \frac{\alpha\Delta T}{h}\frac{L^2}{2}$$

$$\underbrace{\frac{19}{24}\frac{qL^4}{EI} + \frac{qL^4}{6EI} - \frac{qL^4}{EI}\frac{11}{24} + \frac{qL^4}{2EI}}_{qL\left[\frac{19+4-11+12}{24}\right]} = X\left[\frac{2L^3}{3EI} + \frac{L^3}{3EI}\right]$$

$X = qL$ positivo, ok!