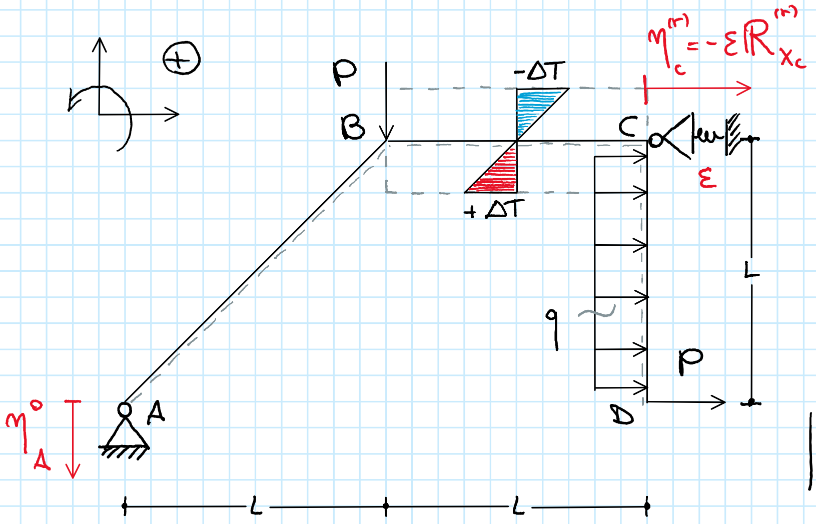
IPER 13

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA DETERMINANDO IL DIAGRAMMA DEI MOMENTI



POSIZIONI

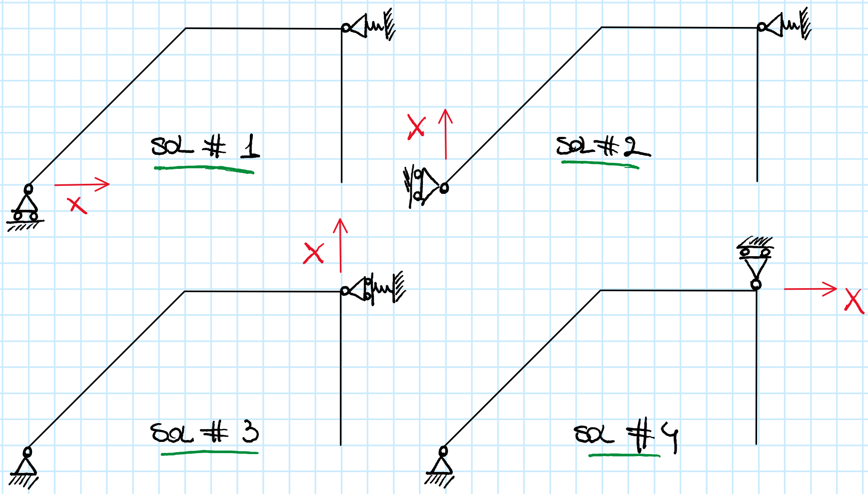
$$|P| = qL$$

$$|\varepsilon| = \frac{L^3}{12EI}\left[\sqrt{2}+1\right]$$

$$|\eta_A^{0}| = \frac{qL^4}{3EI}\left[\sqrt{2}+1\right]$$

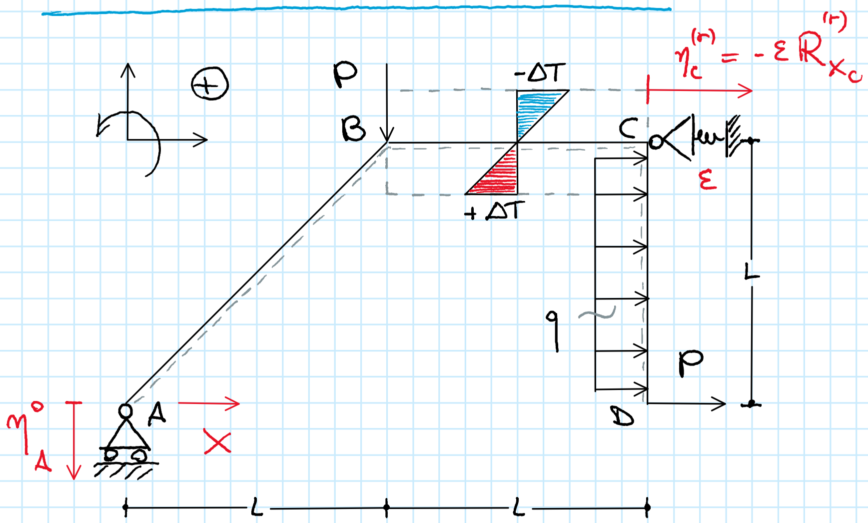
$$\left|\frac{\alpha\,\Delta T}{4h}\right| = -\frac{qL^2}{3EI}\left[\frac{5}{8}\sqrt{2}+1\right]$$

SOLUZIONI POSSIBILI :

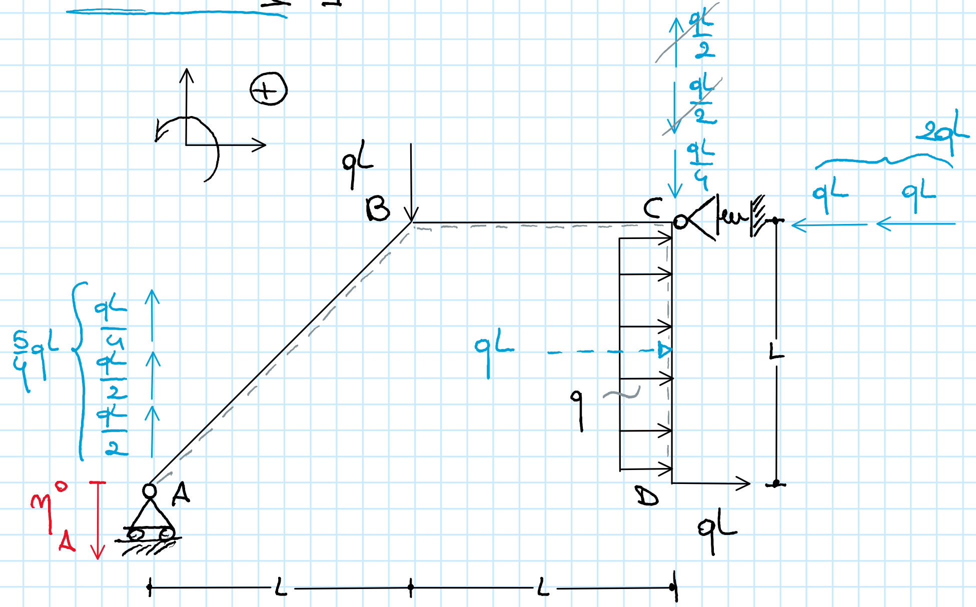


SOLUZIONE # 1

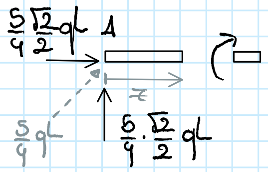
- SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



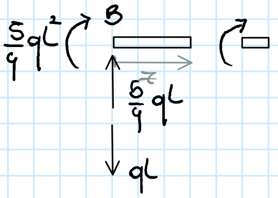
TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



$$\boxed{M^{(0)}(z) = \frac{5}{8}\sqrt{2}\, qL \cdot z}$$

$$\begin{cases} M_A = 0 \\ M_B = \frac{5}{4} qL^2 \end{cases}$$

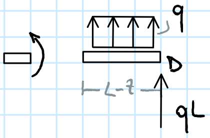
TRATTO BC $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{5}{4} qL^2 + \frac{5}{4} qLz - qLz}$$

$$\begin{cases} M_B = \frac{5}{4} qL^2 \\ M_C = \frac{3}{2} qL^2 \end{cases}$$

TRATTO CD $\quad 0 \le z \le L$



$$M^{(0)}(z) = q\frac{(L-z)^2}{2} + qL(L-z) =$$

$$\boxed{= \frac{qz^2}{2} - 2qLz + \frac{3}{2} qL^2}$$

$$\begin{cases} M_C = \frac{3}{2} qL^2 \\ M_D = \emptyset \end{cases}$$

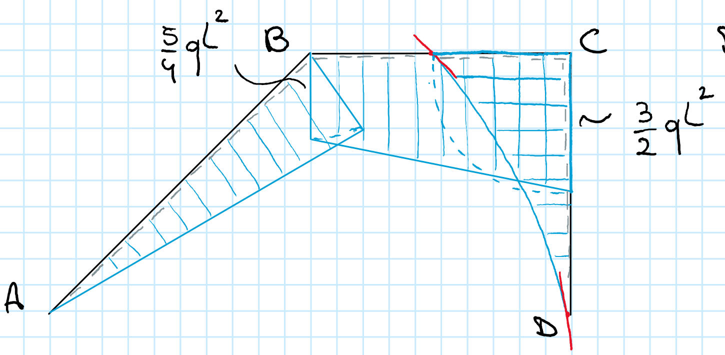
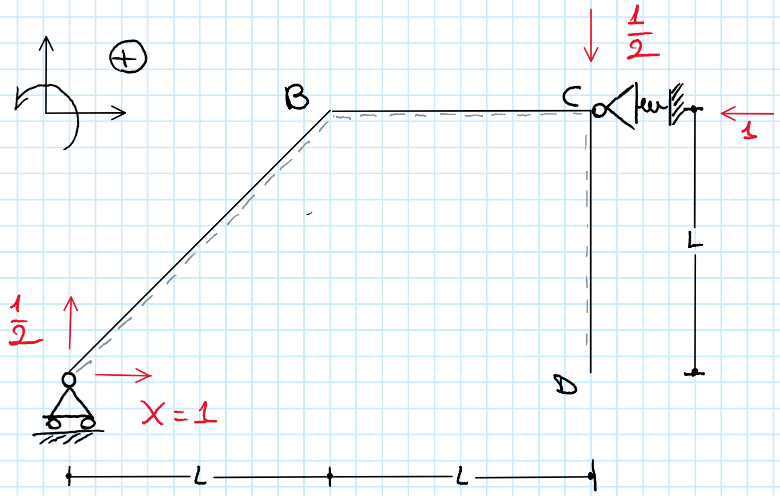
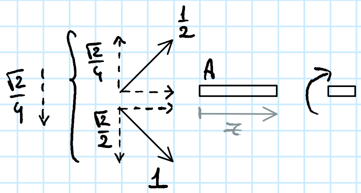


DIAGRAMMA $M^{(0)}(z)$

• SCHEMA [1] SOLO X = 1

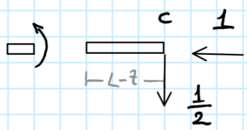


TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = -\frac{\sqrt{2}}{4}\cdot z} \qquad \begin{cases} M_A = 0 \\ M_B = -\frac{L}{2} \end{cases}$$

TRATTO BC $0 \le z \le L$

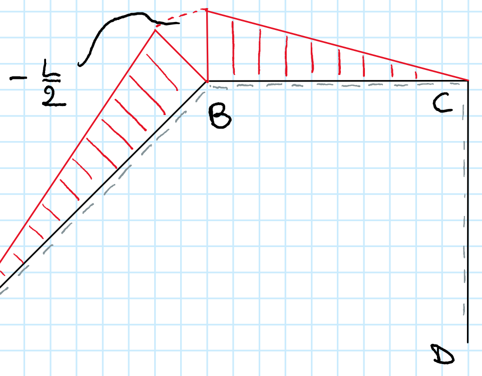


$$\boxed{M^{(1)}(z) = -\frac{(L-z)}{2}} \qquad \begin{cases} M_B = -\frac{L}{2} \\ M_C = \emptyset \end{cases}$$

TRATTO CD $0 \le z \le L$

$$\boxed{M^{(1)}(z) = \emptyset}$$

DIAGRAMMA $M^{(1)}(z)$



$$\mathcal{L}_{ve} = X_i^{f} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)}\, \eta_j^{(r)} =$$

$$= 1 \cdot \eta_A^{(r)} + \frac{1}{2}\left(-\eta_A^{\circ}\right) + \underbrace{R^{(1)}_{x_C}}_{-1} \cdot \underbrace{\eta_C^{(r)}}_{-\varepsilon \underbrace{R^{(r)}_{x_C}}_{R^{(0)}_{x_C} + R^{(1)}_{x_C} X}} - \frac{1}{2} \cdot \emptyset =$$

$$= -\frac{\eta_A^{\circ}}{2} - \varepsilon\left(2qL + X\right)$$

$$\alpha_{ui} = \int_{Str} M^{(1)} \frac{M^{(r)}}{EI}\, dStr + \int_{Str} M^{(f)} \frac{\alpha \overline{\Delta T}}{h}\, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI}\, dStr + \frac{X}{EI} \int_{Str} \left[M^{(1)}\right]^2 dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h}\, dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} M^{(1)} M^{(0)} dz + \int_{BC} M^{(1)} M^{(0)} dz \right\} + \frac{X}{EI} \left\{ \int_{AB} \left[M^{(1)}\right]^2 dz + \int_{BC} \left[M^{(1)}\right]^2 dz \right\} +$$

$$+ \int_{BC} M^{(1)} \frac{\alpha \overline{\Delta T}}{h}\, dStr =$$

$$= \frac{1}{EI} \left\{ \int_0^{\sqrt{2}L} \left[-\frac{\sqrt{2}}{4} z\right] \frac{5}{4} qL\sqrt{2}\, z\, dz + \int_0^L \left[-\frac{(L-z)}{2}\right] \left[\frac{5}{4} qL^2 + \frac{qL}{4} z\right] dz + \right.$$

$$\left. + \frac{X}{EI} \left\{ \int_0^{\sqrt{2}L} \left[-\frac{\sqrt{2}}{4} z\right]^2 + \int_0^L \left[-\frac{(L-z)}{2}\right]^2 dz \right\} + \int_0^L -\frac{(L-z)}{2} \cdot \frac{\alpha \overline{\Delta T}}{h}\, dz = \right.$$

$$+ \frac{1}{EI} \left\{ -\frac{5}{16} qL \left[\frac{z^3}{3}\right]_0^{\sqrt{2}L} + \int_0^L \left[-\frac{5}{8} qL^3 - \frac{qL^2}{8} z + \frac{5}{8} qL^2 z + \frac{qL}{8} z^2\right] dz + \right.$$

$$+ \frac{X}{EI} \left\{ \int_0^{\sqrt{2}L} \frac{z^2}{8} dz + \frac{1}{4} \int_0^L (L-z)^2 dz \right\} - \frac{\alpha \overline{\Delta T}}{2h} \int_0^L (L-z)\, dz =$$

$$= \frac{1}{EI} \left\{ -\frac{5}{16} qL \cdot \frac{L^3 2\sqrt{2}}{3} - \frac{5}{8} qL^3 \left[z\right]_0^L - \frac{qL^2}{8} \left[\frac{z^2}{2}\right]_0^L + \frac{5}{8} qL^2 \left[\frac{z^2}{2}\right]_0^L + \frac{qL}{8} \left[\frac{z^3}{3}\right]_0^L \right\}$$

$$+ \frac{X}{EI} \left\{ \frac{1}{8} \left[\frac{z^3}{3}\right]_0^{\sqrt{2}L} + \frac{L^2}{4} \left[z\right]_0^L + \frac{1}{4} \left[\frac{z^3}{3}\right]_0^L - \frac{L}{2} \left[\frac{z^2}{2}\right]_0^L \right\} + \frac{\alpha \overline{\Delta T}}{2h} \left[\frac{z^2}{2}\right]_0^L +$$

$$- \frac{\alpha \overline{\Delta T}}{2h} L \left[z\right]_0^L =$$

$$= \frac{1}{EI} \left\{ \frac{5}{24} qL^4\sqrt{2} - \frac{5}{8} qL^4 - \frac{qL^4}{16} + \frac{5}{16} qL^4 + \frac{qL^4}{24} \right\} +$$

$$+ \frac{X}{EI} \left\{ \frac{L^3\sqrt{2}}{12} + \frac{L^3}{12} \right\} - \frac{\alpha \overline{\Delta T}}{2h} L^2 + \frac{\alpha \overline{\Delta T}}{4h} L^2 =$$

$$= \frac{1}{EI} \left\{ -\frac{5}{24} qL^4 \sqrt{2} - \frac{1}{3} qL^4 \right\} + \frac{XL^3}{12EI} \left[\sqrt{2} + 1\right] - \frac{\alpha \overline{\Delta T}}{4h} L^2$$

$$= \frac{1}{EI} \left\{ -\frac{5\sqrt{2}}{8} - 1 \right\} + \frac{XL^3}{12EI} \left[\sqrt{2} + 1\right] - \frac{\alpha \overline{\Delta T}}{4h} L^2$$

$\mathcal{L}_{ve} = \mathcal{L}_{vi}$ formica

$$-\frac{M_A^\circ}{2} - \varepsilon\left\{2qL + X\right\} = \frac{qL^4}{3EI}\left\{-\frac{5}{8}\sqrt{2} - 1\right\} + \frac{XL^3}{12EI}\left[\sqrt{2}+1\right] - \frac{\alpha\Delta T}{4h}L^2$$

$$-\frac{M_A^\circ}{2} - \varepsilon\, 2qL - \varepsilon X = \frac{qL^4}{3EI}\left\{-\frac{5}{8}\sqrt{2} - 1\right\} + \frac{XL^3}{12EI}\left[\sqrt{2}+1\right] - \frac{\alpha\Delta T}{4h}L^2$$

$$-\frac{1}{2}\left(\frac{qL^4}{3EI}\left[\sqrt{2}+1\right]\right) - \frac{L^3}{12EI}\left[\sqrt{2}+1\right]\left(2qL + X\right) = \frac{qL^4}{3EI}\left\{\cancel{-\frac{5}{8}\sqrt{2}} - 1\right\} +$$

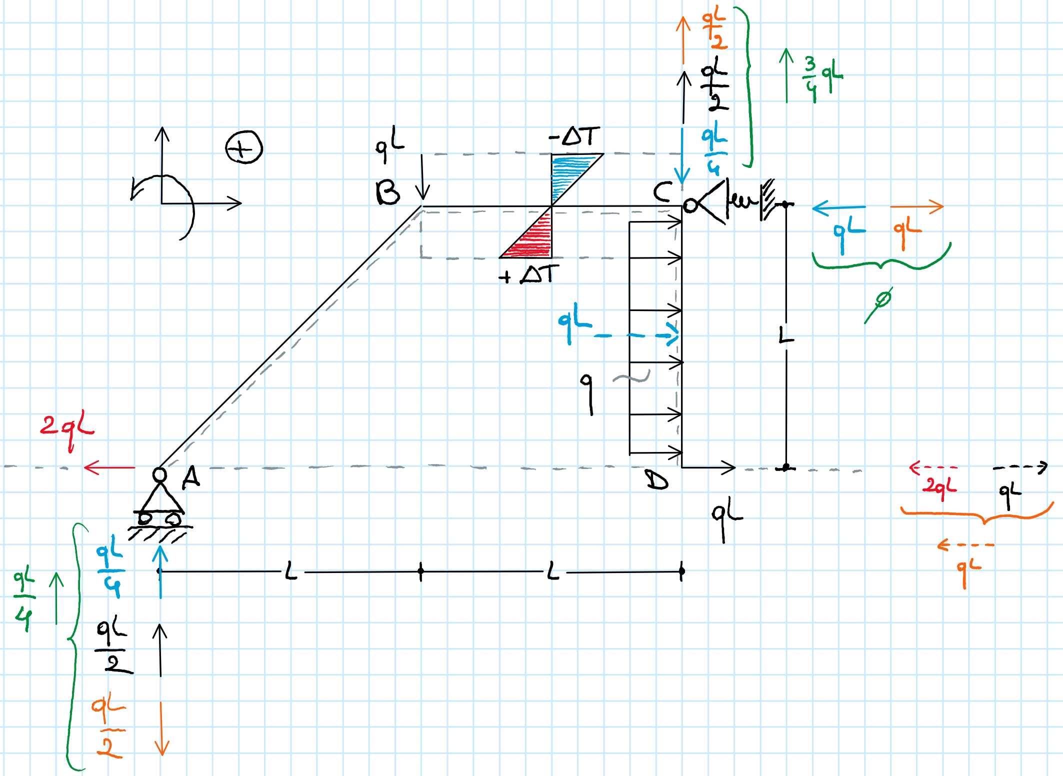
$$+\frac{XL^3}{12EI}\left[\sqrt{2}+1\right] + \frac{qL^4}{3EI}\left\{\cancel{\frac{5}{8}\sqrt{2}} + 1\right\}$$

$$-\frac{1}{6}\frac{qL^4}{EI}\left[\cancel{\sqrt{2}+1}\right] - \frac{1}{6}\frac{qL^4}{EI}\left[\cancel{\sqrt{2}+1}\right] = \frac{1}{12}\frac{XL^3}{EI}\left[\cancel{\sqrt{2}+1}\right] + \frac{1}{12}\frac{XL^3}{EI}\left[\cancel{\sqrt{2}+1}\right]$$

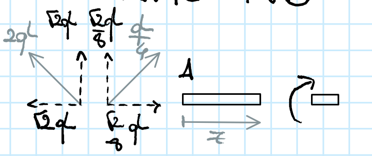
$$-\frac{1}{3}qL = \frac{1}{6}X$$

$X = -2qL$ NEGATIVO, VERSO CONTRARIO

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



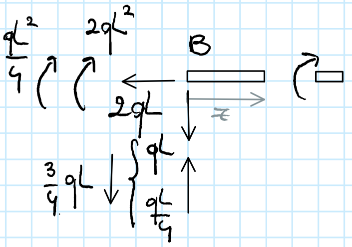
TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



$$\boxed{M^{(r)}(z) = \frac{9}{8}\sqrt{2}\, qLz}$$

$$\begin{cases} M_A = \emptyset \\ M_B = \frac{9}{4} qL^2 \end{cases}$$

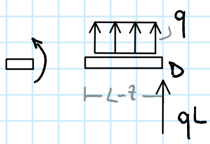
TRATTO BC $\quad 0 \le z < L$



$$\boxed{M^{(r)}(z) = \frac{9}{4} qL^2 - \frac{3}{4} qLz}$$

$$\begin{cases} M_B = \frac{9}{4} qL^2 \\ M_C = \frac{3}{2} qL^2 \end{cases}$$

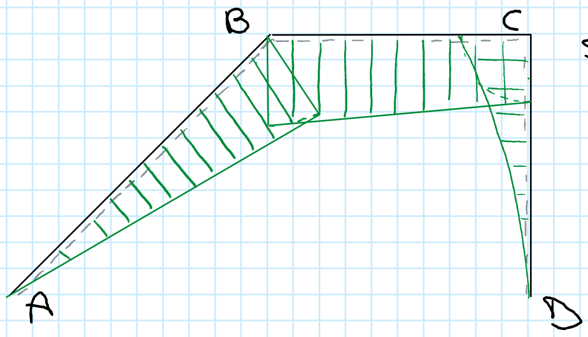
TRATTO CD $\quad 0 \le z \le L$



$$M^{(r)}(z) = qL(L-z) + q\frac{(L-z)^2}{2}$$
$$= qL(L-z) + \frac{q}{2}(L^2 + z^2 - 2Lz)$$
$$= qL^2 - qLz + \frac{qL^2}{2} + \frac{qz^2}{2} - qLz$$
$$= \boxed{\frac{3}{2} qL^2 - 2qLz + \frac{qz^2}{2}}$$

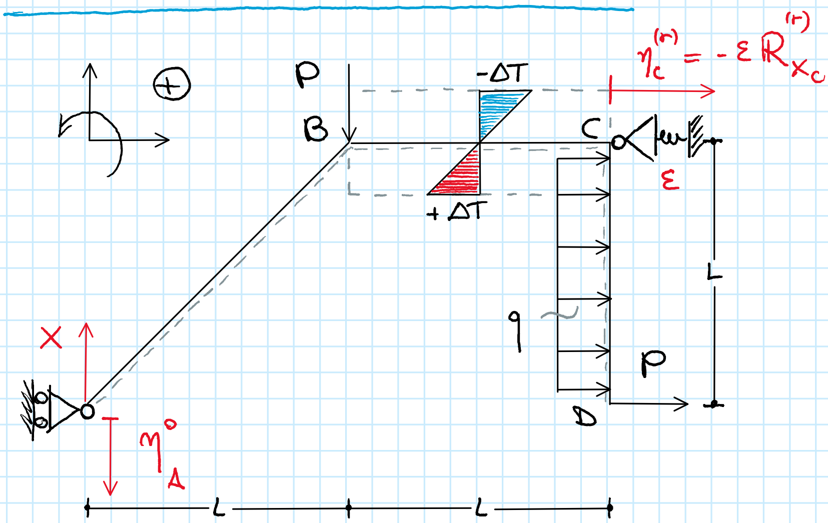
$$\begin{cases} M_C = \frac{3}{2} qL^2 \\ M_D = \emptyset \end{cases}$$

DIAGRAMMA $M^{(r)}(z)$

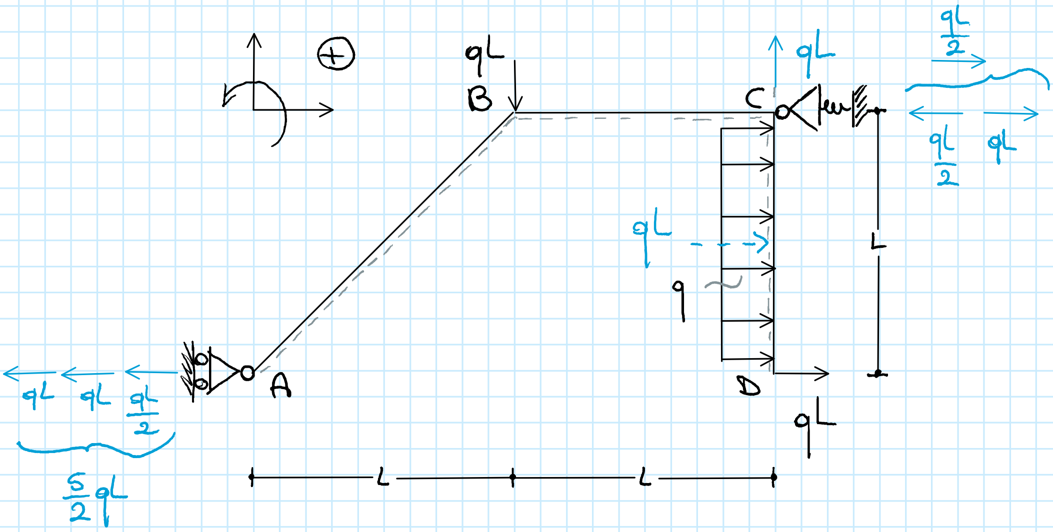


$\boxed{\text{SOLUZIONE \# 2}}$

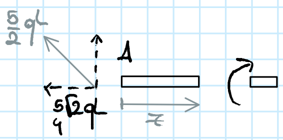
- SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]   SOLO CARICHI ESTERNI



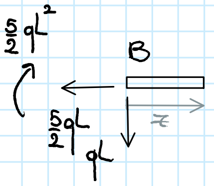
TRATTO AB $0 \le z \le \sqrt{2}L$



$$\boxed{M^{(0)}(z) = \frac{5}{4}\sqrt{2}\,qL\,z}$$

$$\begin{cases} M_A = 0 \\ M_B = \frac{5}{2}qL^2 \end{cases}$$

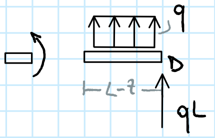
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{5}{2}qL^2 - qLz}$$

$$\begin{cases} M_B = \frac{5}{2}qL^2 \\ M_C = \frac{3}{2}qL^2 \end{cases}$$

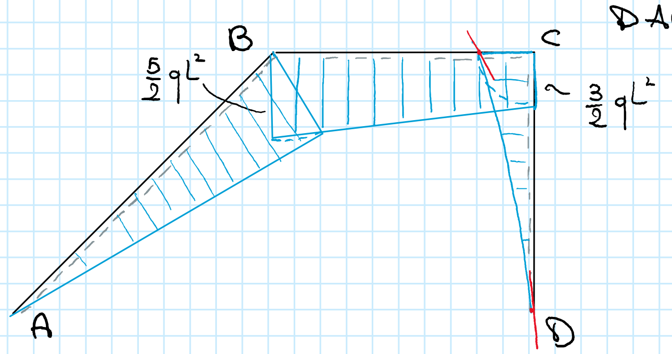
TRATTO CD $0 \le z \le L$



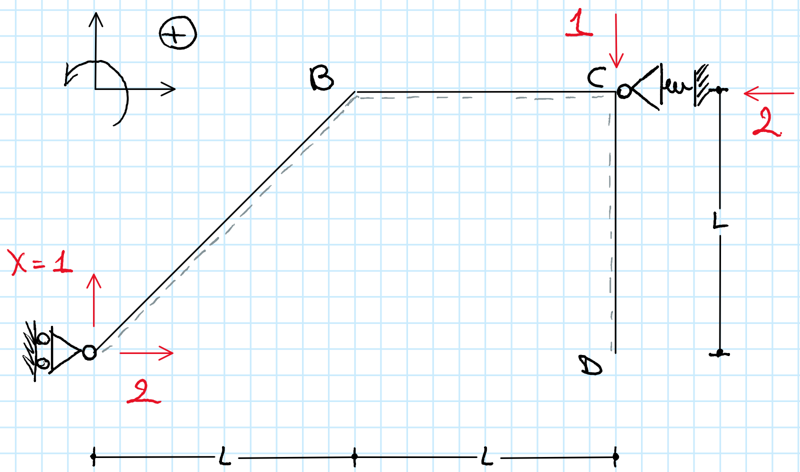
$$\boxed{M^{(0)}(z) = \frac{qz^2}{2} - 2qLz + \frac{3}{2}qL^2}$$

$$\begin{cases} M_C = \frac{3}{2}qL^2 \\ M_D = \emptyset \end{cases}$$

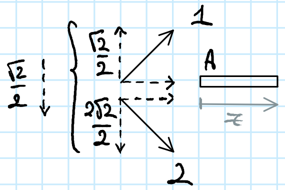
DIAGRAMMA $M^{(0)}(z)$



• SCHEMA [1] SOLO X=1



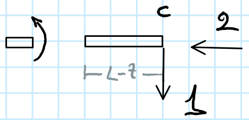
TRATTO AB $0 \le z \le \sqrt{2}L$



$$\boxed{M^{(1)}(z) = -\frac{\sqrt{2}}{2}z}$$

$$\begin{cases} M_A = \emptyset \\ M_B = -L \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -(L-z)}$$

$$\begin{cases} M_B = -L \\ M_C = \emptyset \end{cases}$$

TRATTO CD $0 \leq z \leq L$

$$\boxed{M^{(1)}(z) = \phi}$$

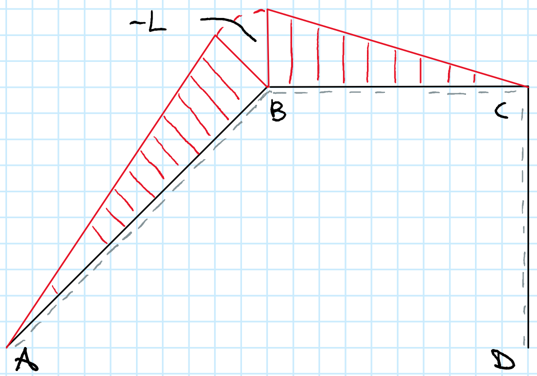


DIAGRAMMA $M^{(1)}z$

$$\alpha_{Ve} = X^{(f)} \cdot \eta_i^{(r)} + \sum_j \mathbb{R}_j^{(f)} \eta_j^{(r)} = 1 \, (-\eta_A^{\circ}) + \underbrace{\mathbb{R}_{X_C}^{(1)}}_{-2} \cdot \underbrace{\eta_C^{(r)}}_{-\varepsilon \mathbb{R}_{X_C}^{(r)}} -$$

$$= \underbrace{-\eta_A^{\circ} + 2\varepsilon \left[\frac{qL}{2} - 2X\right]}$$

$$\underbrace{\mathbb{R}_{X_C}^{(0)}}_{+q\frac{L}{2}} + \underbrace{\mathbb{R}_{X_C}^{(1)}}_{-2} X$$

$$\alpha_{Vi} = \int_{Str} \underset{M^{(1)}}{M^{(f)}} \frac{\overset{M^{(0)}+M^{(1)}}{M^{(r)}}}{EI} dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + X \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} \left[-\frac{\sqrt{2}}{2} z\right] \left[\frac{5}{4} qL\sqrt{2} \cdot z\right] dz + \int_{BC} (z-L) \left[\frac{5}{2} qL^2 - qLz\right] dz \right\}$$

$$+ \frac{X}{EI} \left\{ \int_{AB} \left[-\frac{\sqrt{2}}{2} z\right]^2 dz + \int_{BC} (L-z)^2 dz \right\} + \int_{BC} -(L-z) \alpha \frac{\Delta T}{h} dz =$$

$$= \frac{1}{EI} \left\{ \int_0^{\sqrt{2}L} -\frac{5}{4} qL z^2 dz + \int_0^L \left[\frac{5}{2} qL^2 z - qLz^2 - \frac{5}{2} qL^3 + qL^2 z\right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^{\sqrt{2}L} \frac{z^2}{2} dz + \int_0^L \left[L^2 - z^2 - 2Lz\right] \right\} + \frac{\alpha \Delta T}{h} \int_0^L (z-L) dz =$$

$$= \frac{1}{EI}\left\{-\frac{5}{4}qL\left[\frac{z^3}{3}\right]_0^{\sqrt{2}L} + L^3\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - \frac{5}{2}qL^3\left[z\right]_0^L + \right.$$

$$\left. + qL^2\left[\frac{z^2}{2}\right]_0^L\right\} + \frac{X}{EI}\left\{\frac{1}{2}\left[\frac{z^3}{3}\right]_0^{\sqrt{2}L} + L^3\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{L}\right]_0^L\right\} +$$

$$+ \frac{\alpha\Delta T}{h}\left\{\left[\frac{z^2}{2}\right]_0^L - L\left[z\right]_0^L\right\} =$$

$$= \frac{1}{EI}\left\{-\frac{5}{6}\sqrt{2}qL^4 + \frac{5}{4}qL^4 - \frac{qL^4}{3} - \frac{5}{2}qL^4 + \frac{qL^4}{2}\right\} + \frac{X}{EI}\left\{\frac{1}{3}\sqrt{2}L^3 + L^3 + \right.$$

$$\left. + \frac{L^3}{3} - 2\frac{L^3}{2}\right\} + \alpha\frac{\Delta T}{h}\left\{\frac{L^2}{2} - L^2\right\} =$$

$$\underbrace{= \frac{1}{EI}qL^4\left\{-\frac{5}{6}\sqrt{2} - \frac{13}{12}\right\} + \frac{XL^3}{EI}\left\{\sqrt{2} + 1\right\} - \frac{\alpha\Delta T}{h}\frac{L^2}{2}}$$

$\mathcal{L}_{ve} = \mathcal{L}_{vi}$ fornisce

$$-\eta_A^\circ + 2\varepsilon\left[\frac{qL}{2} - 2X\right] = \frac{5}{6}\sqrt{2}\frac{qL^4}{EI} - \frac{13}{12}\frac{qL^4}{EI}\left[\sqrt{2}+1\right] - \frac{\alpha\Delta T}{h}\frac{L^2}{2}$$

$$-\frac{qL^4}{3EI}\left[\sqrt{2}+1\right] + \frac{qL^4}{12EI}\left[\sqrt{2}+1\right] = -\frac{5}{6}\sqrt{2}\frac{qL^4}{EI} - \frac{13}{12}\frac{qL^4}{EI} + \frac{XL^3}{3EI}\left[\sqrt{2}+1\right] +$$

$$+\frac{4}{6}\frac{qL^4}{EI}\left\{\frac{5}{4}\sqrt{2} + 1\right\}$$

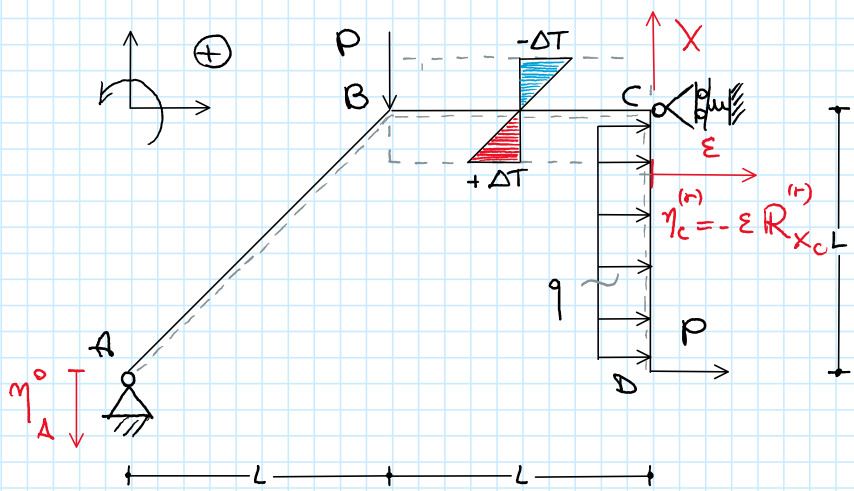
$$-\frac{3}{12}\sqrt{2}qL^4 - \frac{3}{12}qL^4 + \frac{5}{6}\sqrt{2}qL^4 + \frac{13}{12}qL^4 - \frac{20}{24}\sqrt{2}qL^4 - \frac{4}{6}qL^4 = X\left[\sqrt{2}+1\right]\left\{\frac{L^3}{3} + \frac{L^3}{3}\right\}$$

$$\sqrt{2}qL^4\left\{\frac{3}{12} + \frac{10}{12} - \frac{5}{12}\right\} + qL^4\left\{-\frac{3}{12} + \frac{13}{12} - \frac{8}{12}\right\} = X\frac{2}{3}L^3\left[\sqrt{2}+1\right]$$

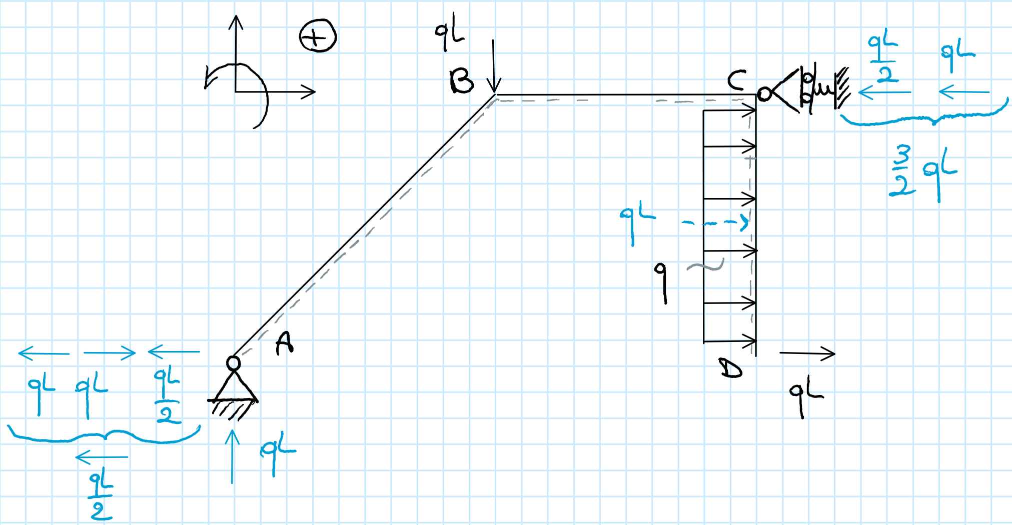
$$\frac{qL^4}{6}\left[\sqrt{2}+1\right] = XL^3\frac{2}{3}\left[\sqrt{2}+1\right] \qquad X = \frac{qL}{6}\cdot\frac{3}{2} = \frac{qL}{4}$$ POSITIVO, ok

SOLUZIONE # 3

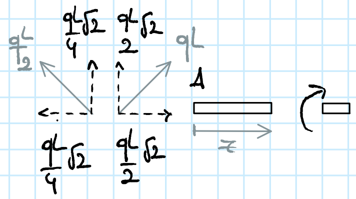
• SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



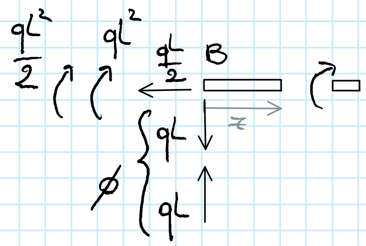
TRATTO AB $0 \le z \le \sqrt{2}L$



$$\boxed{M^{(0)}(z) = \frac{3}{4}\sqrt{2}\, qL\, z}$$

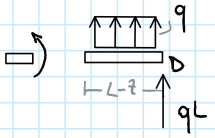
$$\begin{cases} M_A = \emptyset \\ M_B = \frac{3}{2} qL^2 \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{3}{2} qL^2}$$ cost.

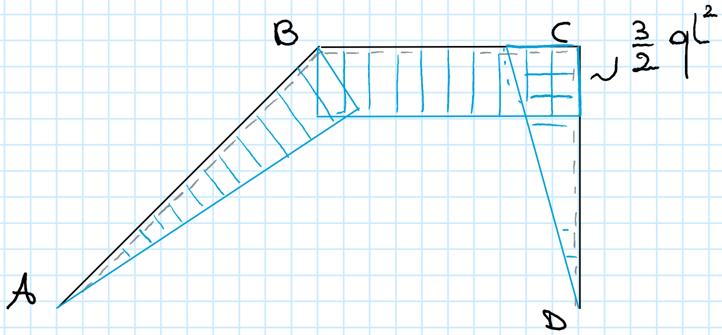
TRATTO CD $\quad 0 \leq z \leq L$



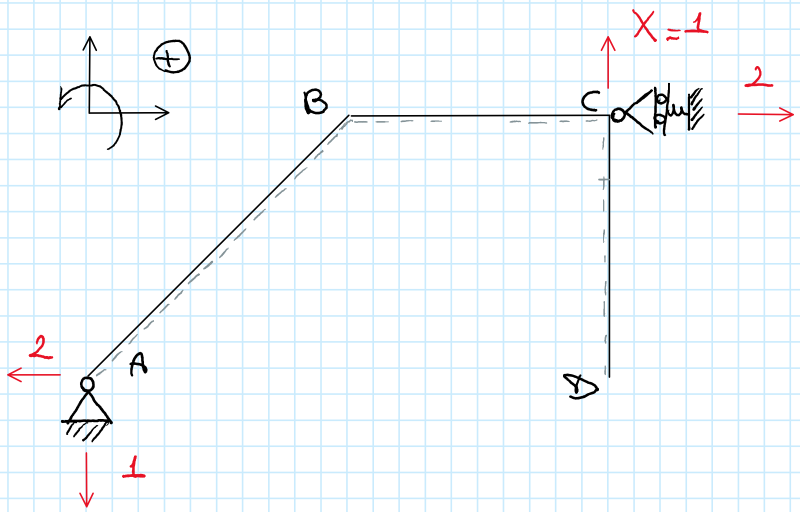
$$M^{(0)}(z) = qL(L-z) + q\,\frac{(L-z)^2}{2}$$
$$= qL(L-z) + \frac{q}{2}(L^2 + z^2 - 2Lz)$$
$$= qL^2 - qLz + \frac{qL^2}{2} + \frac{qz^2}{2} - qLz$$
$$= \boxed{\frac{3}{2}qL^2 - 2qLz + \frac{qz^2}{2}}$$

$$\begin{cases} M_C = \frac{3}{2}qL^2 \\ M_D = \emptyset \end{cases}$$

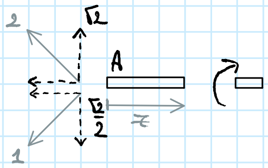
DIAGRAMMA $M^{(0)}(z)$



• SCHEMA [1] $\quad$ SOLO $X = 1$



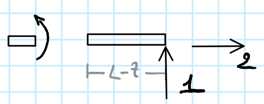
TRATTO AB $\quad 0 \leq z \leq \sqrt{2}L$



$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{2}z}$$

$$\begin{cases} M_A = 0 \\ M_B = L \end{cases}$$

TRATTO BC $\quad 0 \leq z \leq L$

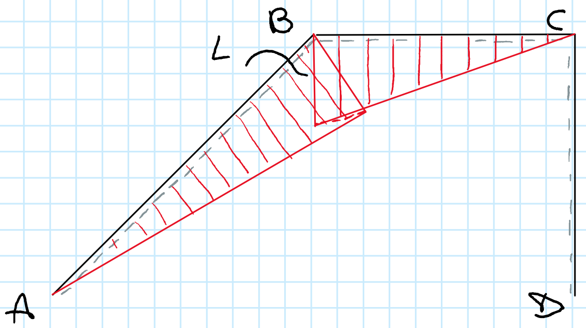


$$\boxed{M^{(1)}(z) = L - z}$$

$$\begin{cases} M_B = L \\ M_C = \emptyset \end{cases}$$

TRATTO CD $0 \leq z \leq L$

$$\boxed{M^{(1)}(z) = \emptyset}$$



$$\underbrace{L_{ve}} = X^{(f)}_i \cdot \eta^{(r)}_i + \sum_j R^{(f)}_j \; \eta^{(r)}_j =$$

$$= 1 \cdot \emptyset + (1)\,(\eta^{\circ}_A) + \underbrace{R^{(1)}_{X_C}}_{2} \cdot \underbrace{\eta^{(r)}_C}_{-\varepsilon R^{(r)}_{X_C}}$$

$$\underbrace{R^{(r)}_{X_C}} = \underbrace{R^{(0)}_{X_C}}_{-\frac{3}{2}qL} + \underbrace{R^{(1)}_{X_C}}_{2} X$$

$$= \underbrace{\eta^{\circ}_A - 2\varepsilon\left[-\frac{3}{2}qL + 2X\right]}$$

$$\underbrace{L_{vi}} = \int_{Str} \underbrace{M^{(f)}}_{M^{(1)}} \frac{\overbrace{M^{(r)}}^{M^{(0)} + M^{(1)}X}}{EI} \, dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} \, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} \, dStr + X \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} \, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} \, dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} \left[\frac{\sqrt{2}}{2} z\right] \left[\frac{3}{4}\sqrt{2}\, qLz\right] dz + \int_{BC} \left[L - z\right] \left[\frac{3}{2} qL^2\right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_{AB} \left[\frac{\sqrt{2}}{2} z\right]^2 dz + \int_{BC} (L - z)^2 \, dz \right\} + \int_{BC} (L - z) \frac{\alpha \Delta T}{h} \, dz +$$

$$+ \frac{1}{EI} \left\{ \int_0^{\sqrt{2}L} \frac{3}{4} qL z^2 \, dz - \int_0^L \left[\frac{3}{2} qL^3 - \frac{3}{2} qL^2 z\right] dz \right\} + \frac{X}{EI} \left\{ \int_0^{\sqrt{2}L} \frac{z^2}{2} \, dz + \right.$$

$$\left. + \int_0^L (L^2 + z^2 - 2zL) \, dz \right\} + \int_0^L L \frac{\alpha \Delta T}{h} \, dz - \int_0^L z \frac{\alpha \Delta T}{h} \, dz =$$

$$\frac{1}{EI}\left\{\frac{3}{4}qL\left[\frac{z^3}{3}\right]_0^{\sqrt{2}L}+\frac{3}{2}qL^3\left[z\right]_0^L-\frac{3}{2}qL^2\left[\frac{z^2}{2}\right]_0^L\right\}+$$

$$+\frac{X}{EI}\left\{\frac{1}{2}\left[\frac{z^3}{3}\right]_0^{\sqrt{2}L}+L^2\left[z\right]_0^L+\left[\frac{z^3}{3}\right]_0^L-2L\left[\frac{z^2}{2}\right]_0^L\right\}+\frac{\alpha\Delta T}{h}\left\{L\left[z\right]_0^L-\left[\frac{z^2}{2}\right]_0^L\right\}-$$

$$=\frac{1}{EI}\left\{\frac{\sqrt{2}}{2}qL^4+\frac{3}{2}qL^4-\frac{3}{4}qL^4\right\}+\frac{X}{EI}\left\{\frac{\sqrt{2}}{3}L^3+L^3+\frac{L^3}{3}-L^3\right\}+$$

$$+\frac{\alpha\Delta T}{h}\left\{L^2-\frac{L^2}{2}\right\}=$$

$$\underbrace{+\frac{1}{EI}\left\{\frac{\sqrt{2}}{2}qL^4+\frac{3}{4}qL^4\right\}+\frac{X}{EI}\left\{\frac{\sqrt{2}}{3}L^3-\frac{L^3}{3}\right\}+\frac{\alpha\Delta T}{h}\cdot\frac{L^2}{2}}$$

$\mathcal{L}_{ve}-\mathcal{L}_{vi}$ fornisce

$$\eta_A^\circ-2\varepsilon\left[-\frac{3}{2}qL+2X\right]=\frac{1}{EI}\left\{\frac{\sqrt{2}}{2}qL^4+\frac{3}{4}qL^4\right\}+\frac{X}{EI}\left\{\frac{\sqrt{2}}{3}L^3+\frac{L^3}{3}\right\}+\frac{L^2}{2}\frac{\alpha\Delta T}{h}$$

$$\frac{X}{EI}\left\{\frac{L^3}{3}(\sqrt{2}+1)+4\varepsilon\right\}=-\frac{qL^4}{EI}\left\{\frac{\sqrt{2}}{2}+\frac{3}{4}\right\}-\frac{\alpha\Delta T}{h}\frac{L^2}{2}+\eta_A^\circ+3\varepsilon qL$$

$$\frac{X}{EI}\left\{\frac{L^3}{3}(\sqrt{2}+1)+\frac{\cancel{4}}{\cancel{12}_{3}}L^3(\sqrt{2}+1)\right\}=-\frac{qL^4}{EI}\left\{\frac{\sqrt{2}}{2}+\frac{3}{4}\right\}+\frac{\cancel{4}^{2}}{\cancel{6}_{3}}\frac{qL^4}{EI}\left\{\frac{5}{8}\sqrt{2}+1\right\}+$$

$$+\frac{1}{3}\frac{qL^4}{EI}(\sqrt{2}+1)+\frac{\cancel{3}^{1}}{\cancel{12}_{4}}\frac{qL^4}{EI}(\sqrt{2}+1)$$

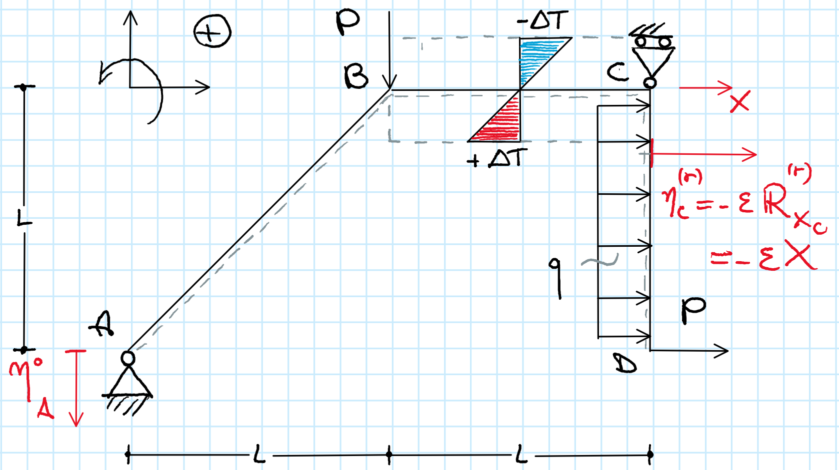
$$\frac{XL^3}{EI}\left\{\frac{2}{3}(\sqrt{2}+1)\right\}=\frac{qL^4}{EI}\left\{-\frac{\sqrt{2}}{2}-\frac{3}{4}+\frac{2}{3}+\frac{5}{12}\sqrt{2}+\frac{\sqrt{2}}{3}+\frac{1}{3}+\frac{\sqrt{2}}{4}+\frac{1}{4}\right\}$$

$$\frac{XL^3}{EI}\left\{\frac{2}{3}(\sqrt{2}+1)\right\}=\frac{qL^4}{EI}\left\{\frac{1}{2}(\sqrt{2}+1)\right\}$$

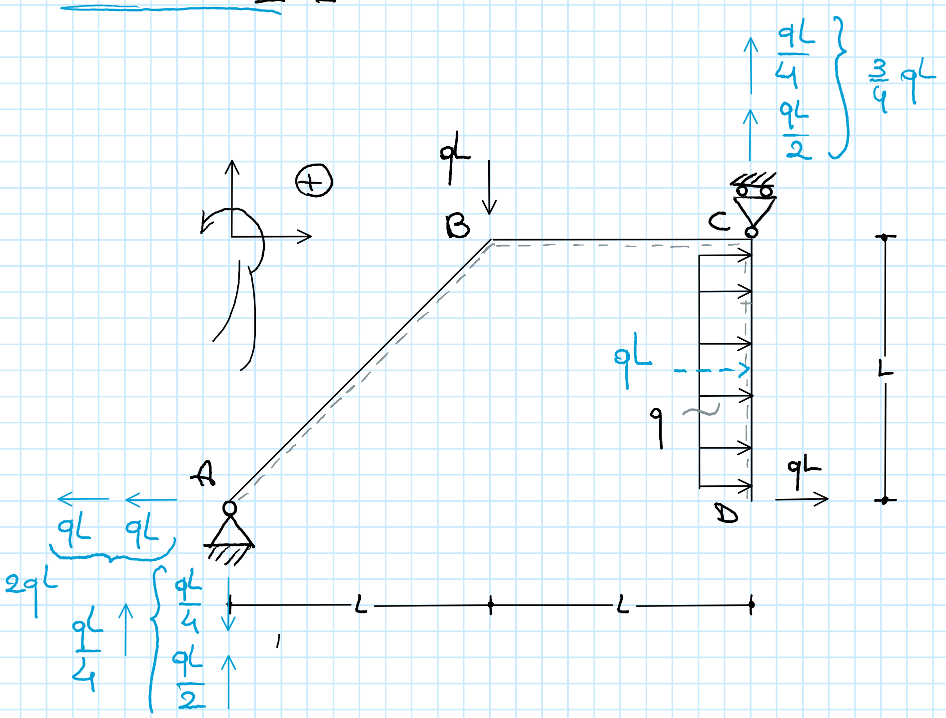
$$X=\frac{1}{2}\cdot\frac{3}{2}qL=\frac{3}{4}qL \quad \text{POSITIVO}$$

SOLUZIONE # 4

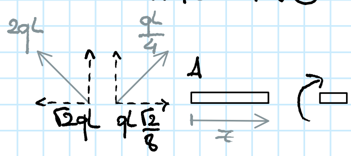
- SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



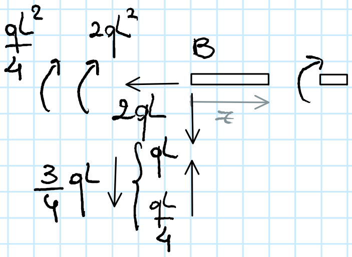
TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



$$\boxed{M^{(0)}(z) = \frac{9}{8}\sqrt{2}\,qL\,z}$$

$$\begin{cases} M_A = 0 \\ M_B = \frac{9}{4}qL^2 \end{cases}$$

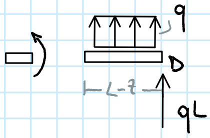
TRATTO BC $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{9}{4}qL^2 - \frac{3}{4}qLz}$$

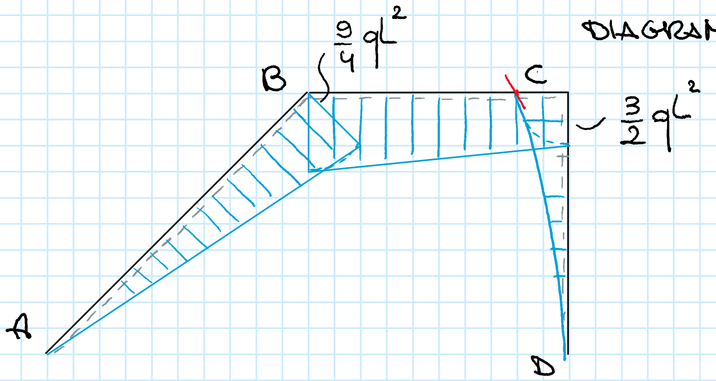
$$\begin{cases} M_B = \frac{9}{4}qL^2 \\ M_C = \frac{3}{2}qL^2 \end{cases}$$

TRATTO CD $\quad 0 \le z \le L$

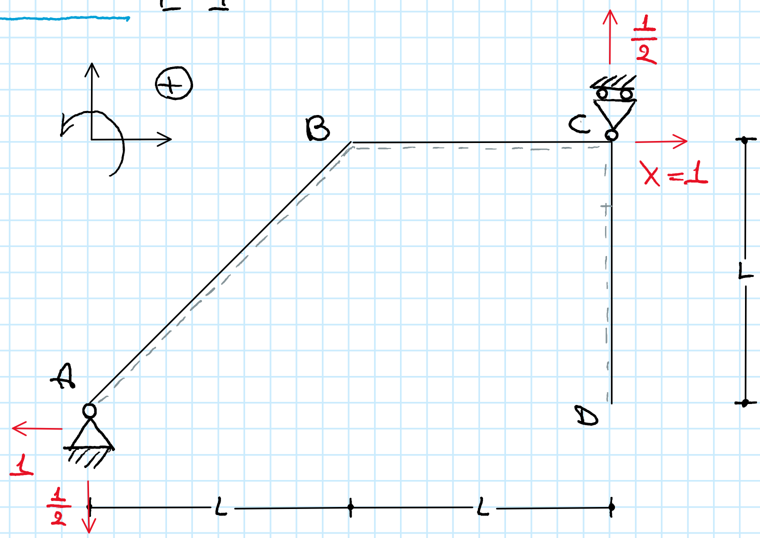


$$M^{(0)}(z) = qL(L-z) + q\frac{(L-z)^2}{2}$$
$$= qL(L-z) + \frac{q}{2}(L^2+z^2-2Lz)$$
$$= qL^2 - qLz + \frac{qL^2}{2} + \frac{qz^2}{2} - qLz$$
$$= \boxed{\frac{3}{2}qL^2 - 2qLz + \frac{qz^2}{2}} \qquad \begin{cases} M_C = \frac{3}{2}qL^2 \\ M_D = \emptyset \end{cases}$$

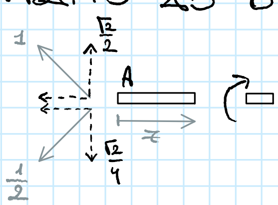
DIAGRAMMA $M^{(0)}(z)$



• SCHEMA [1] SOLO $X=1$

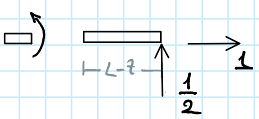


TRATTO AB $\quad 0 \le z \le \sqrt{2}L$



$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{4}z} \qquad \begin{cases} M_A = \emptyset \\ M_B = \frac{L}{2} \end{cases}$$

TRATTO BC $\quad 0 \le z \le L$

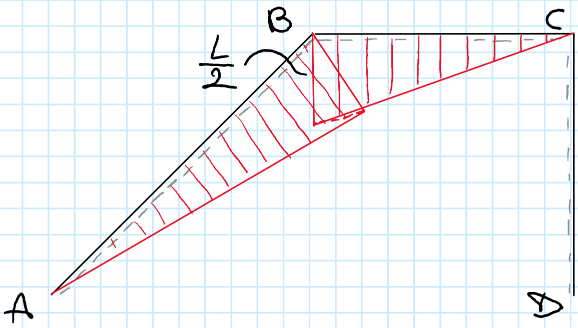


$$\boxed{M^{(1)}(z) = \frac{(L-z)}{2}} \qquad \begin{cases} M_B = \frac{L}{2} \\ M_C = \emptyset \end{cases}$$

TRATTO CD $0 \leq z \leq L$

$$\boxed{M^{(1)}(z) = \emptyset}$$

DIAGRAMMA $M^{(1)}(z)$



$$\underbrace{\mathcal{L}_{ve}} = X_1 \eta_1^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{\eta_c^{(r)}}_{\varepsilon X} - \frac{1}{2}\left(-\eta_A^{\circ}\right) = \underbrace{\varepsilon X + \frac{\eta_A^{\circ}}{2}}$$

$$\underbrace{\mathcal{L}_{vi}} = \int_{Str} M^{(f)} \frac{M^{(1)}}{EI} + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + X \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} dStr \quad \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \frac{1}{EI}\left\{ \int_{AB} \underbrace{\left[\frac{\sqrt{2}}{4} z\right]\left[\frac{9}{8}\sqrt{2}\, qLz\right]}^{\frac{9}{16} qLz^2} dz - \int_{BC} \frac{(L-z)}{2}\left[\frac{9}{4} qL^2 - \frac{3}{4} qLz\right] dz \right\} +$$

$$+ \frac{X}{EI}\left\{ \int_{AB} \left[\frac{\sqrt{2}}{4} z\right]^2 dz + \int_{BC} \left[\frac{L-z}{2}\right]^2 dz \right\} + \int_{BC} \frac{L-z}{2} \; \frac{\alpha \Delta T}{h} dz =$$

$$= \frac{1}{EI}\left\{ \int_0^{\sqrt{2}L} \frac{9}{16} qLz^2 dz + \int_0^L \left(\frac{9}{8} qL^3 \underbrace{- \frac{9}{8} qL^2 z - \frac{3}{8} qL^2 z}_{-\frac{3}{2} qL^2 z} + \frac{3}{8} qLz^2\right) dz \right\} +$$

$$+ \frac{X}{EI}\left\{ \int_0^{\sqrt{2}L} \frac{z^2}{8} dz + \int_0^L \frac{1}{4}\left[L^2 + z^2 - 2Lz\right] dz \right\} + \frac{\alpha \Delta T}{2h} \int_0^L (L-z) dz =$$

$$= \frac{1}{EI}\left\{ \frac{9}{16} qL \left[\frac{z^3}{3}\right]_0^{\sqrt{2}L} + \frac{9}{8} qL^3 \left[z\right]_0^L - \frac{3}{2} qL^2 \left[\frac{z^2}{2}\right] + \frac{3}{8} qL \left[\frac{z^3}{3}\right]_0^L \right\} +$$

$$+\frac{X}{EI}\left\{\frac{1}{8}\left[\frac{z^3}{3}\right]_0^{\sqrt{2}L}+\frac{L^2}{4}\left[z\right]_0^L+\frac{1}{4}\left[\frac{z^3}{3}\right]_0^L-\frac{L}{2}\left[\frac{z^2}{2}\right]_0^L\right\}+\frac{\alpha\Delta T}{2h}\left\{L\left[z\right]_0^L-\left[\frac{z^2}{2}\right]_0^L\right\}=$$

$$+\frac{1}{EI}\left\{\frac{3}{8}\sqrt{2}qL^4+\frac{9}{8}qL^4-\frac{3}{4}qL^4+\frac{1}{8}qL^4\right\}+\frac{X}{EI}\left\{\frac{\sqrt{2}}{12}L^3+\frac{L^3}{4}+\frac{L^3}{12}+\right.$$

$$\left.-\frac{L^3}{4}\right\}+\frac{\alpha\Delta T}{2h}\left\{L^2-\frac{L^2}{2}\right\}=$$

$$=\frac{1}{EI}\left\{\frac{3}{8}\sqrt{2}qL^4+\frac{1}{2}qL^4\right\}+\frac{X}{EI}\left\{\frac{\sqrt{2}}{12}L^3-\frac{L^3}{12}\right\}+\frac{\alpha\Delta T}{2h}\left\{\frac{L^2}{2}\right\}$$

$\mathcal{L}_{ve}-\mathcal{L}_{vi}$ formisca

$$-\varepsilon X+\frac{M_A^0}{2}=\frac{1}{EI}\left\{\frac{3}{8}\sqrt{2}qL^4+\frac{1}{2}qL^4\right\}+\frac{X}{EI}\left\{\frac{\sqrt{2}}{12}L^3+\frac{L^3}{12}\right\}+\frac{\alpha\Delta T}{2h}\left\{\frac{L^2}{2}\right\}$$

$$-\frac{1}{12}\frac{XL^3}{EI}\left[\sqrt{2}+1\right]+\frac{1}{6}\frac{qL^4}{EI}\left[\sqrt{2}+1\right]=\frac{qL^4}{EI}\left[\frac{3}{8}\sqrt{2}+\frac{1}{2}\right]$$

$$+\frac{1}{12}\frac{XL^3}{EI}\left[\sqrt{2}+1\right]-\frac{qL^4}{3EI}\left[\frac{5}{8}\sqrt{2}+1\right]$$

$$+\frac{1}{6}\frac{XL^3}{EI}\left[\sqrt{2}+1\right]=\frac{qL^4}{EI}\left[\frac{\sqrt{2}}{6}+\frac{1}{6}-\frac{3}{8}\sqrt{2}-\frac{1}{2}+\frac{5}{24}\sqrt{2}+\frac{1}{3}\right]$$

$$+\frac{1}{6}\frac{XL^3}{EI}\left[\sqrt{2}+1\right]\quad\frac{qL^4}{EI}\left[\emptyset\right]$$

$$X=\emptyset \quad \text{ok!}$$