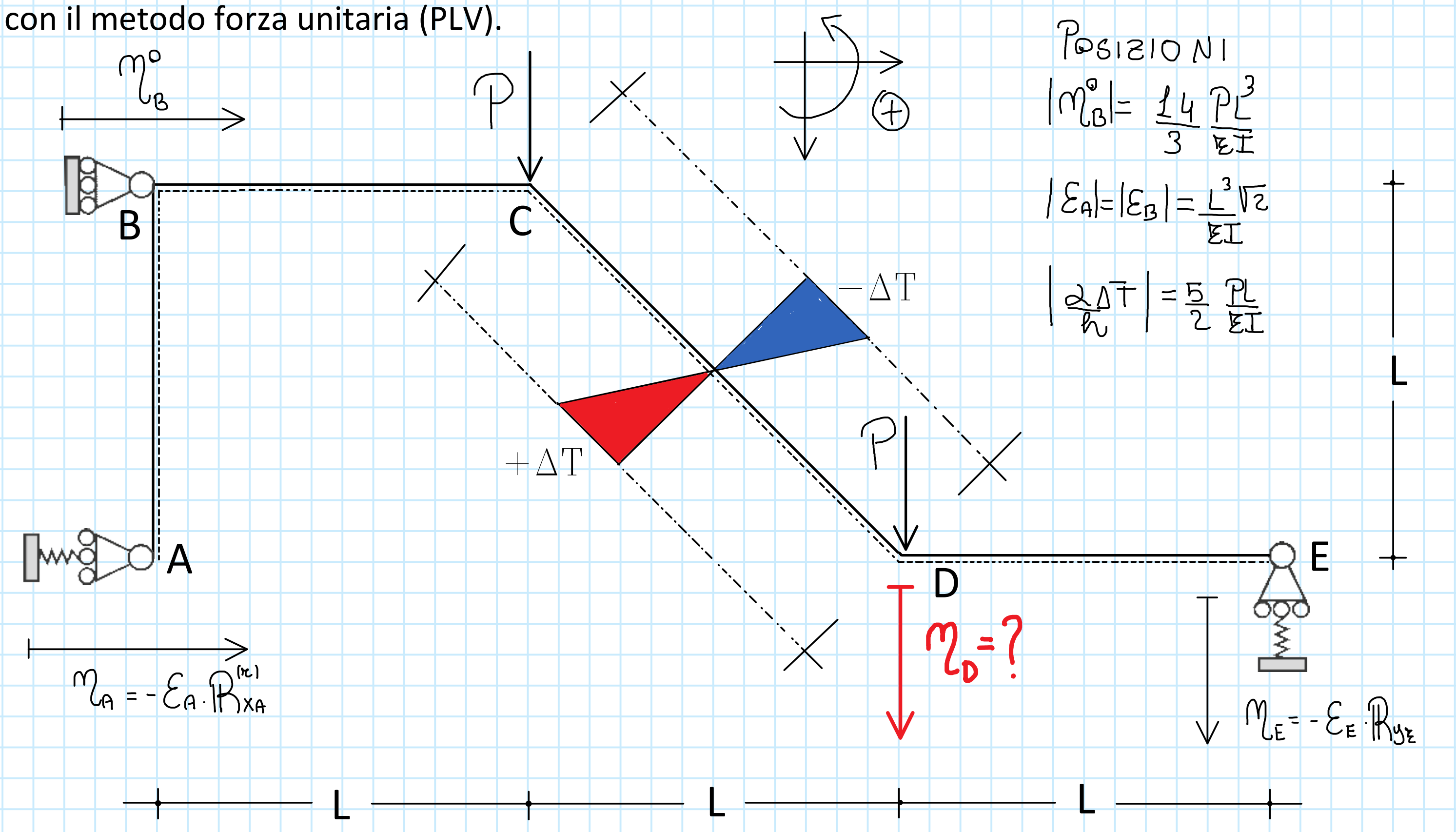
MECCANICA DELLE STRUTTURE - Corso P. FUSCHI Prova scritta

Quesito N. 2

Determinare lo spostamento verticale della sezione D della struttura isostatica seguente con il metodo forza unitaria (PLV).



POSIZIONI

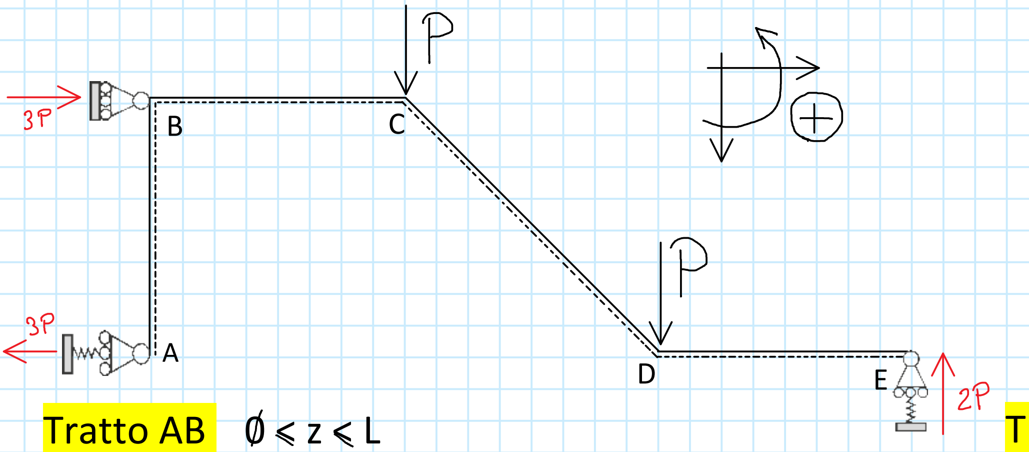
$$|\eta^0_B| = \frac{14}{3}\frac{PL^3}{EI}$$

$$|\varepsilon_A| = |\varepsilon_B| = \frac{L^3\sqrt{2}}{EI}$$

$$\left|\frac{2\alpha\Delta T}{h}\right| = \frac{5}{2}\frac{PL}{EI}$$

SOLUZIONE:

Struttura Reale sulla quale si valutano gli spostamenti generalizzati reali e le CD congruenti espressi tramite le CS associate ai carichi reali!



$\sum F_x = 0 \quad -R_{xA} + R_{xB} = 0 \qquad R_{xA} = +3P$

$\sum F_y = 0 \quad P + P - R_{yE} = 0 \qquad R_{yE} = 2P$

$\sum M_E = 0 \quad P\cancel{L} + 2P\cancel{L} - R_{xB}\cancel{L} = 0 \qquad R_{xB} = 3P$

Tratto AB $0 \leqslant z \leqslant L$

3P, z

$\boxed{M_z^{(rc)} = 3P \cdot z}$ $\quad M_A = 0$, $M_B = 3PL$

Tratto BC $0 \leqslant z \leqslant L$

3PL, z

$\boxed{M_z^{(rc)} = 3PL}$ cost.

Tratto CD $0 \leqslant z \leqslant L\sqrt{2}$

$\frac{\sqrt{2}}{2}P$, 3P, 3PL, 3P, z

$\boxed{M_z^{(rc)} = 3PL - P\frac{\sqrt{2}}{2} \cdot z}$ $\quad M_C = 3PL$, $M_D = 2PL$

Tratto DE $0 \leqslant z \leqslant L$

2P, L - z

$\boxed{M_z^{(rc)} = 2P(L-z)}$ $\quad M_D = 2PL$, $M_E = 0$

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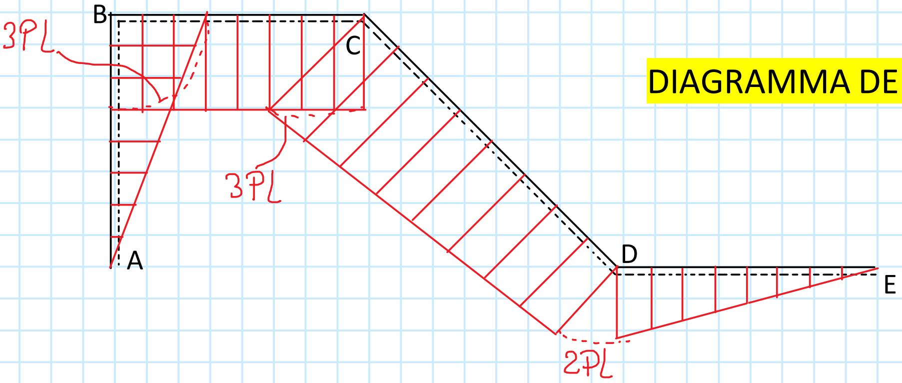


DIAGRAMMA DEI MOMENTI REALI

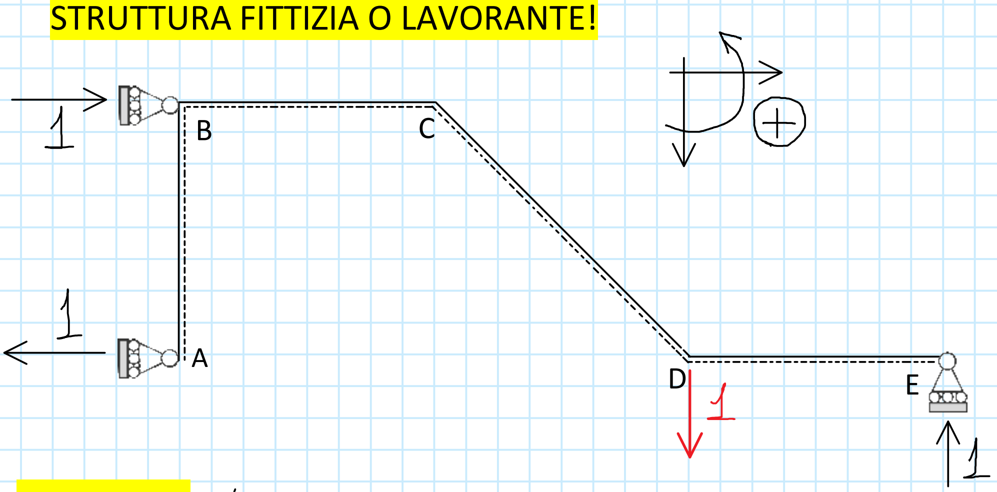
Per calcolare lo spostamento orizzontale della sezione C si assume come sistema fittizio o lavorante quello riportato in figura seguente, cioè quello in cui la struttura in esame è caricata da una forza unitaria applicata in C e diretta verso destra!

STRUTTURA FITTIZIA O LAVORANTE!

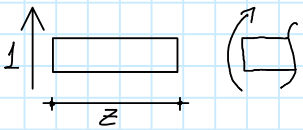
$\sum F_x = 0 \quad -R_{xA} + R_{xB} = 0 \quad R_{xA} = 1$

$\sum F_y = 0 \quad -1 + R_{yE} = 0 \quad R_{yE} = 1$

$\sum M_E = 0 \quad 1L - R_{xB} \cdot L = 0 \quad R_{xB} = 1$

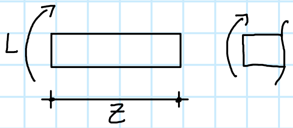


Tratto AB $0 \leq z \leq L$



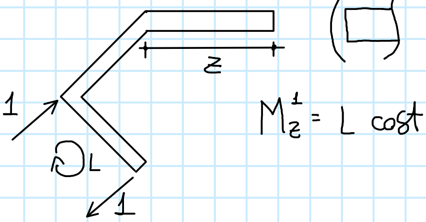
$M_z^1 = z \quad \begin{cases} M_A = 0 \\ M_B = L \end{cases}$

Tratto BC $0 < z < L$



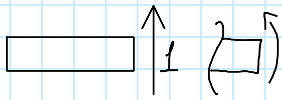
$M_z^1 = L \text{ cost}$

Tratto CD $0 \leq z \leq L\sqrt{2}$

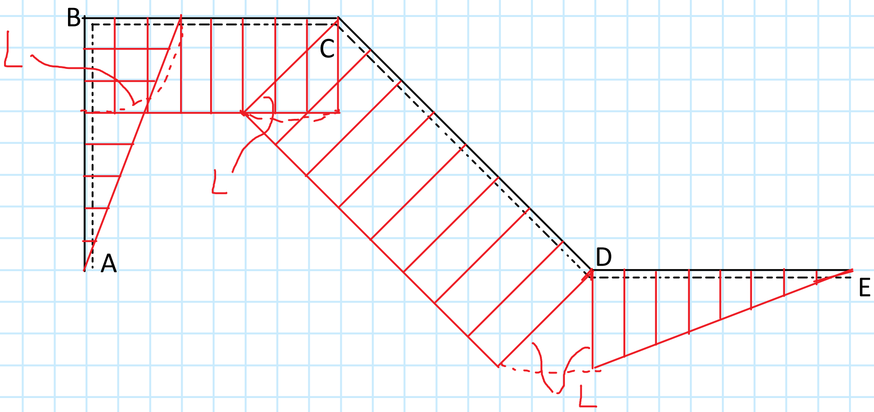


$M_z^1 = L \text{ cost}$

Tratto DE $0 \leq z \leq L$



$M_z^1 = 1(L - z) \quad \begin{cases} M_D = L \\ M_E = 0 \end{cases}$



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APPLICO IL PLV-mdFU nella forma Lve = Lvi può scriversi:

$$L_{ve} = \sum_i F_i^{(\ell)} \cdot \eta_i^{(r)} + \sum_J R_J^{(\ell)} \cdot \eta_J^{(r)} = 1 \cdot \eta_D + R_{x_B}^1 \cdot \eta_B^o + R_{x_A}^1 \cdot \eta_A^r + R_{y_E}^1 \cdot \eta_E^r =$$

$$= 1 \cdot \eta_D + 1\, \eta_B^o - 1\left[-\varepsilon_A(-3P)\right] - 1\left[-\varepsilon_E(-2P)\right] =$$

$$\boxed{L_{ve} = \eta_D + \eta_B^o - \varepsilon_A 3P - \varepsilon_E 2P}$$

$$L_{vi} = \int_{Str} M^{(\ell)}\left[\frac{M^{(r)}}{EI} + \frac{\alpha \Delta T}{h}\right] d_{str} = \int_{str} M^{(\ell)} \frac{M^{(r)}}{EI} + M^{(\ell)} \cdot \frac{\alpha\Delta T}{h}\, d_{str} =$$

$$= \left\{\int_0^L z(3P z)\, dz + \int_0^L L(3PL)\, dz + \int_0^{L\sqrt{2}} L\left(3PL - \frac{P\sqrt{2}}{2} \cdot z\right) dz + \int_0^L 2P(L-z)^2 dz + \int_0^{L\sqrt{2}} L \frac{\alpha\Delta T}{h}\, dz =\right.$$

$$= \frac{1}{EI}\left\{\int_0^L 3Pz^2\, dz + \int_0^L 3PL^2\, dz + \int_0^{L\sqrt{2}} 3PL^2 - \frac{\sqrt{2}}{2}PLz\, dz + \int_0^L \overbrace{2PL^2 + 2Pz^2 - 4PLz}^{2P(L^2+z^2-2LP)}\, dz\right. + \frac{\alpha\Delta T}{h}\int_0^{L\sqrt{2}} L\, dz =$$

$$= \frac{1}{EI}\left\{\left[\frac{3Pz^3}{3}\right]_0^L + \left[3PL^2 z\right]_0^L + \left[3PL^2 z - \frac{\sqrt{2}}{2}PL\frac{z^2}{2}\right]_0^{L\sqrt{2}} + \left[2PL^2 z + \frac{2Pz^3}{3} - \frac{4}{2}PL z^2\right]_0^L\right. + \frac{\alpha\Delta T}{h}\left[zL\right]_0^{L\sqrt{2}} =$$

$$= \frac{1}{EI}\left\{PL^3 + 3PL^3 + 3\sqrt{2}PL^3 - \frac{\sqrt{2}}{2}PL^3 + \cancel{2PL^3} + \frac{2}{3}PL^3 - \cancel{2PL^3}\right\} + \frac{\alpha\Delta T}{h}L^2\sqrt{2} =$$

$$= \frac{PL^3}{EI}\left[4 + \frac{2}{3} + \frac{5}{2}\sqrt{2}\right] + \frac{\alpha\Delta T}{h}L^2\sqrt{2} = \frac{PL^3}{EI}\left[\frac{14}{3} + \frac{5}{2}\sqrt{2}\right] + \frac{\alpha\Delta T}{h}L^2\sqrt{2} =$$

$$\boxed{L_{vi} = \frac{PL^3}{EI}\left[\frac{14}{3} + \frac{5}{2}\sqrt{2}\right] + \frac{\alpha\Delta T}{h}L^2\sqrt{2}}$$

$$L_{ve} = L_{vi}$$

$$\eta_D + \eta_B^o - \varepsilon_A 3P - \varepsilon_E 2P = \frac{PL^3}{EI}\left[\frac{14}{3} + \frac{5}{\sqrt{2}}\right] + \frac{\alpha\Delta T}{h}L^2\sqrt{2}$$

$$\eta_D + \frac{14}{3}\frac{PL^3}{EI} - L^3\sqrt{2}\frac{3P}{EI} - L^3\sqrt{2}\frac{2P}{EI} = \frac{PL^3}{EI}\left[\frac{14}{3} + \frac{5}{2}\sqrt{2}\right] + \frac{5\sqrt{2}}{2}PL^3$$

$$\eta_D + \frac{14}{\cancel{3}}\frac{PL^3}{EI} - L^3\sqrt{2}\frac{3P}{EI} - L^3\sqrt{2}\frac{2P}{EI} = \cancel{\frac{PL^3}{EI}\frac{14}{3}} + \frac{5}{2}\sqrt{2}\frac{PL^3}{EI} + \frac{5}{2}\sqrt{2}PL^3$$

$$\eta_D - 5\sqrt{2}\frac{PL^3}{EI} = 5\sqrt{2}\frac{PL^3}{EI}$$

$$\eta_D = 10\sqrt{2}\frac{PL^3}{EI}$$

POSITIVO!!!
Verso ipotizzato correttamente

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