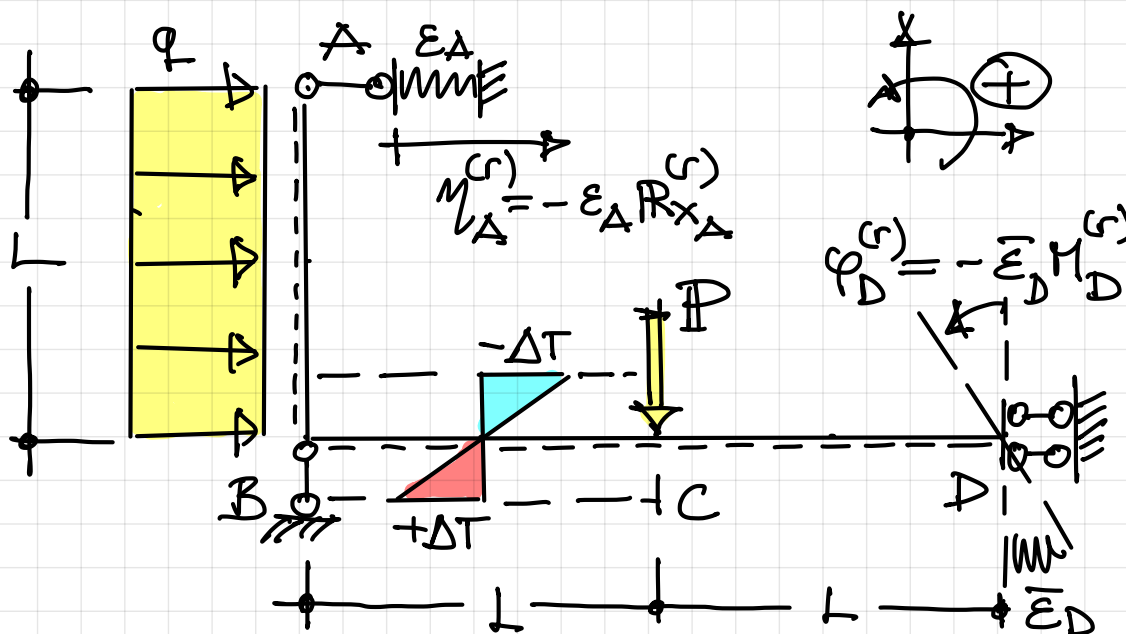


MECCANICA delle STRUTTURE - P. FUSCHI

PROVA SCRITTA del 29 GENNAIO 2025

**TIPO
1**

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI



Posizioni:

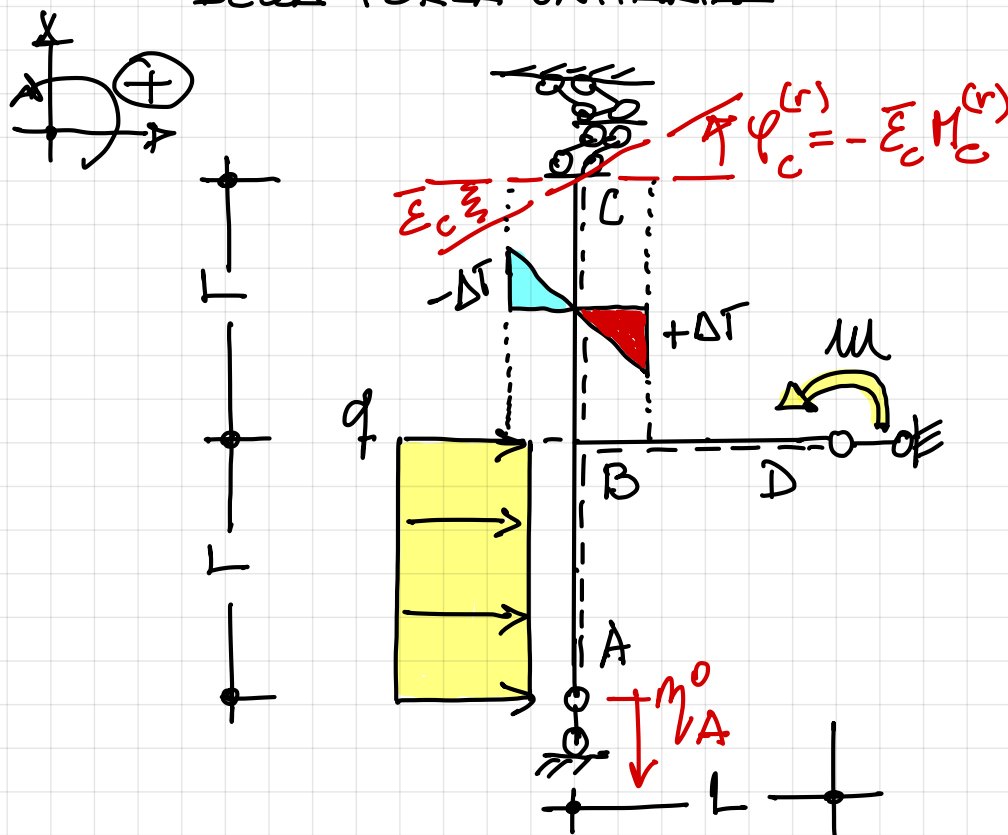
$$|H| = qL$$

$$|E_A| = \frac{5}{3} \frac{L^3}{EI}$$

$$|\bar{E}_D| = \frac{2L}{EI}$$

$$\left| \frac{\alpha \Delta T}{h} \right| = \frac{3qL^2}{8EI}$$

ES. #2 CALCOLARE LO SPOST. VERTICALE DELLA SEZ. D DELLA STRUTTURA SEGUENTE CON IL METODO DELLA FORZA UNITARIA



POSIZIONI:

$$|M| = qL^2$$

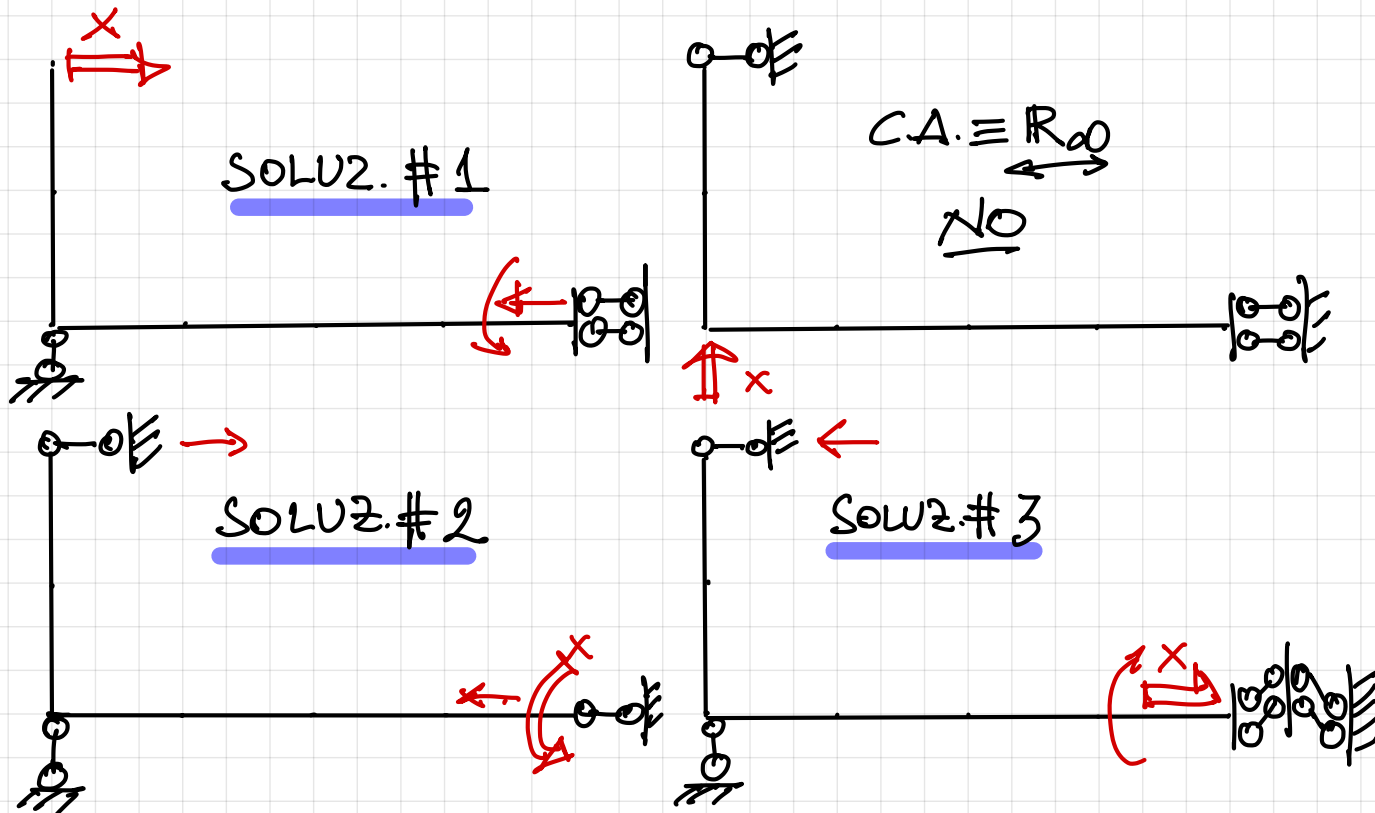
$$|\bar{E}_C| = \frac{L}{EI}$$

$$|M_A^0| = \frac{qL^4}{2EI}$$

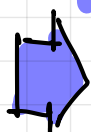
$$\left| \frac{\alpha \Delta T}{h} \right| = \frac{2qL^2}{EI}$$



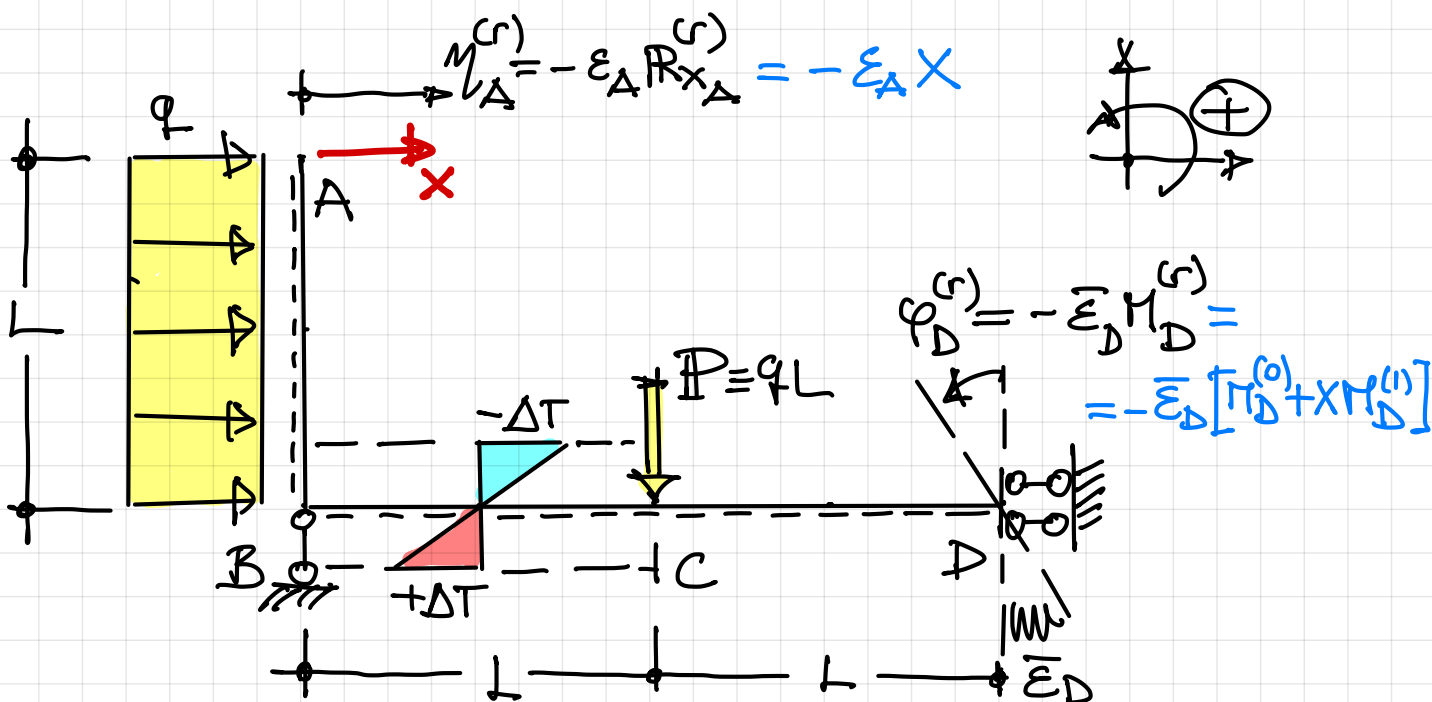
SOLUZIONI POSSIBILI



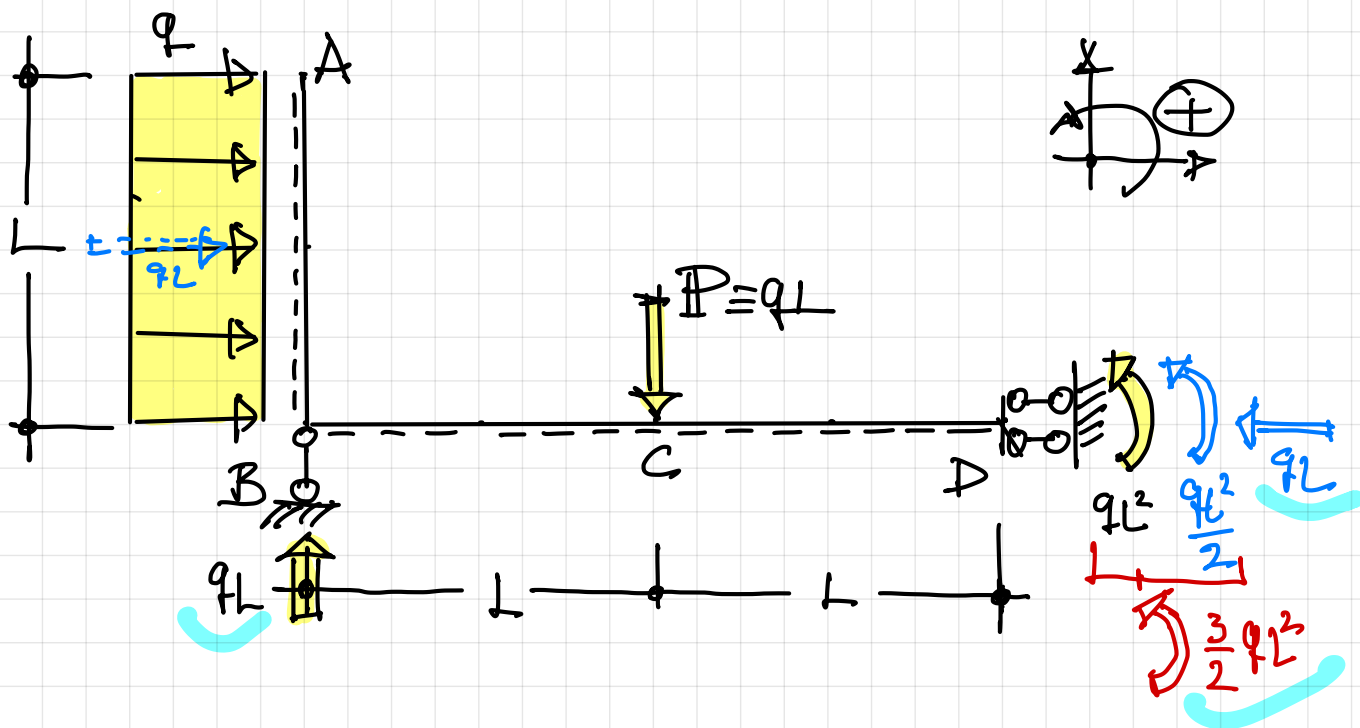
SOLUZIONE #1



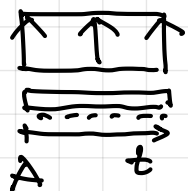
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [o] - SOLO CARICHI ESTERNI



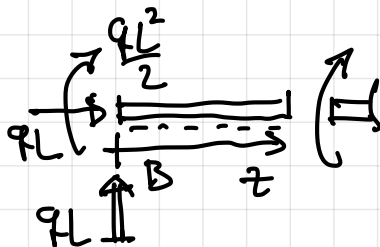
TRATTO AB $0 \leq z \leq L$



$$M^{(0)}(z) = \frac{qz^2}{2}$$

$$\begin{cases} M_A = 0 \\ M_B = \frac{qL^2}{2} \end{cases}$$

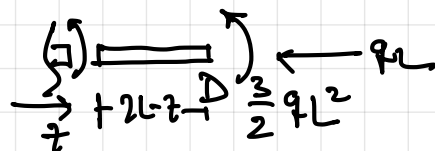
TRATTO BC $0 \leq z \leq L$



$$M^{(0)}(z) = \frac{qL^2}{2} + qL \cdot z$$

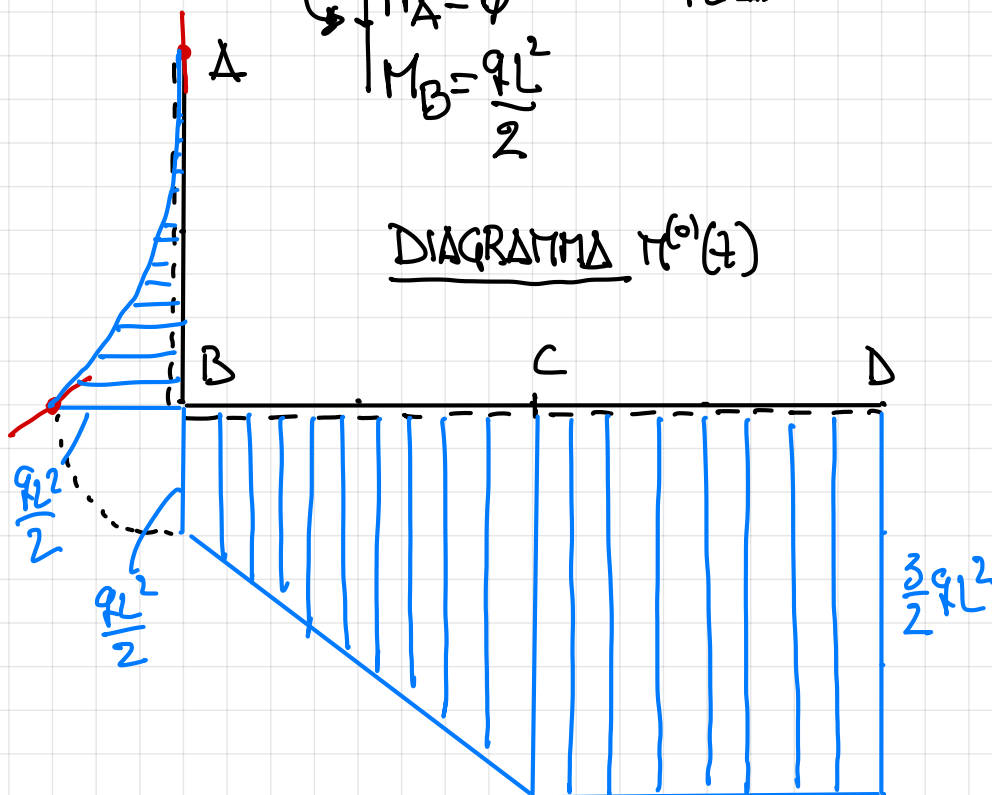
$$\begin{cases} M_B = \frac{qL^2}{2} \\ M_C = \frac{3}{2}qL^2 \end{cases}$$

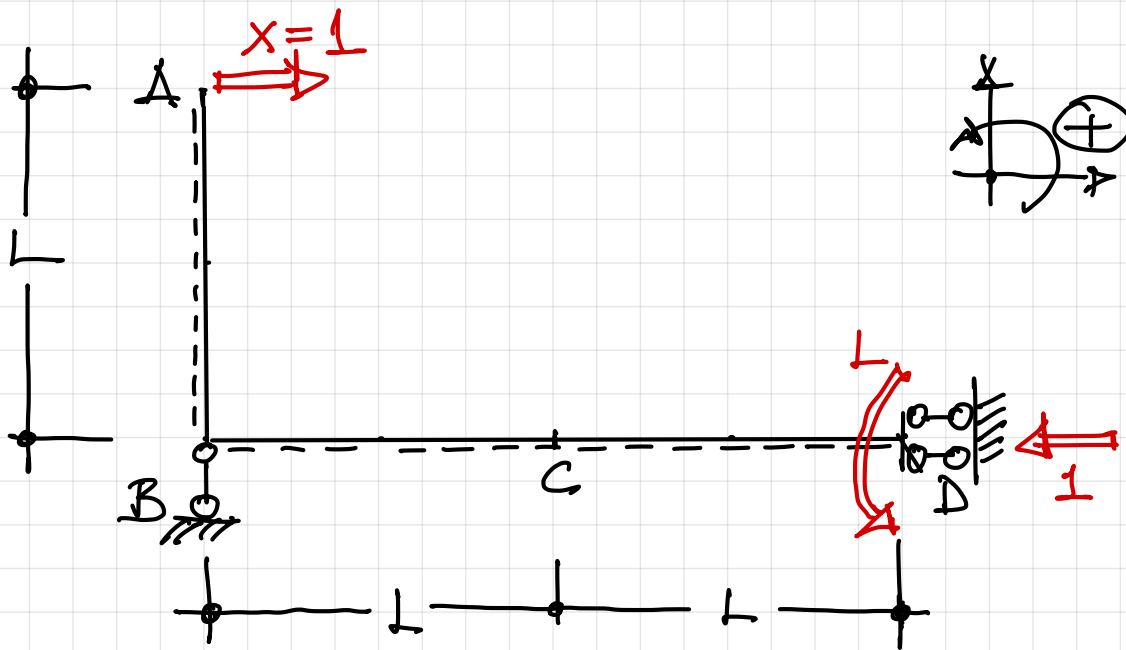
TRATTO CD $L \leq z \leq 2L$



$$M^{(0)}(z) = \frac{3}{2}qL^2 \text{ cost.}$$

DIAGRAMMA $M^{(0)}(z)$





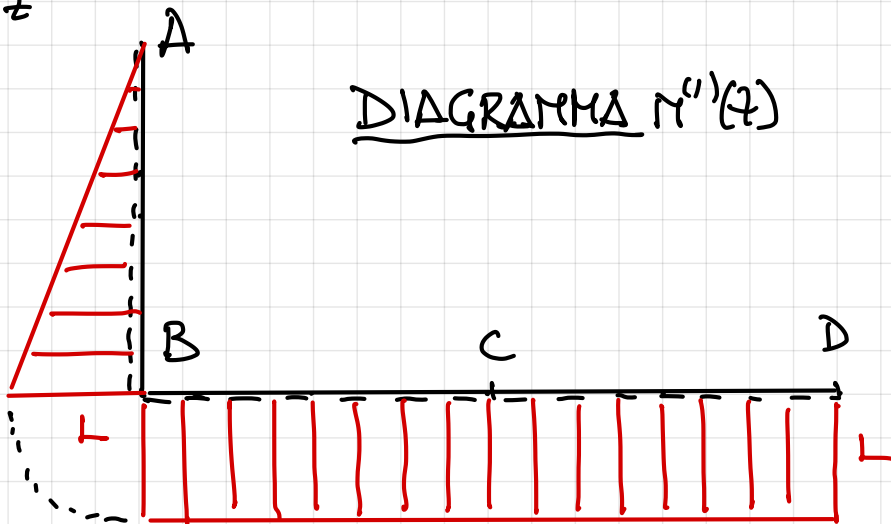
TRATTO AB $0 \leq z \leq L$

$\uparrow 1$
 $A \xrightarrow{z}$
 $\left(\begin{array}{l} M^{(1)}(z) = z \\ M_A = 0 \\ M_B = L \end{array} \right)$

TRATTO BD $0 \leq z \leq 2L$

$\xrightarrow{z} B \xrightarrow{z}$
 $\left(\begin{array}{l} M^{(1)}(z) = L \\ \text{costante} \end{array} \right)$

DIAGRAMMA $M^{(1)}(z)$



$\Rightarrow L_{ve} = \sum_i X_i \eta_i^{(r)} + \sum_j R_j^{(0)} \eta_j^{(r)} = -\varepsilon_A X + \underbrace{M_D^{(1)}}_L (-\varepsilon_D) \left[\underbrace{M_D^{(0)}}_{\frac{3}{2}qL^2} + X \underbrace{M_D^{(1)}}_L \right] =$
 $= -\varepsilon_A X - \varepsilon_D L \left[\frac{3}{2}qL^2 + LX \right]$

$$\begin{aligned}
 \Rightarrow L_{vi} &= \int_{str} M^{(1)} \frac{M^{(2)}}{EI} dstr + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} dstr = \\
 &= \frac{1}{EI} \int_{str} M^{(1)} M^{(2)} dstr + \frac{\alpha}{EI} \int_{str} [M^{(1)}]^2 dstr + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} dstr = \\
 &= \frac{1}{EI} \left[\int_0^L (z) \frac{qz^2}{2} dz + \int_0^L L \left[\frac{qL^2}{2} + qL \cdot z \right] dz + \int_L^{2L} L \left[\frac{3}{2} qL^2 \right] dz \right] + \\
 &\quad + \frac{\alpha}{EI} \left[\int_0^L z^2 dz + \int_0^{2L} L^2 dz \right] + \frac{\alpha \Delta T}{h} \int_0^L L dz = \\
 &= \frac{1}{EI} \left\{ \frac{q}{2} \left[\frac{z^4}{4} \right]_0^L + \frac{qL^3}{2} [z]_0^L + qL^2 \left[\frac{z^2}{2} \right]_0^L + \frac{3}{2} qL^3 [z]_L^{2L} \right\} + \\
 &\quad + \frac{\alpha}{EI} \left\{ \left[\frac{z^3}{3} \right]_0^L + L^2 [z]_0^{2L} \right\} + \frac{\alpha \Delta T}{h} \cdot L [z]_0^L = \\
 &= \frac{1}{EI} \left\{ \frac{q}{8} L^4 + \frac{qL^4}{2} + \frac{qL^2 \cdot L^2}{2} + \frac{3}{2} qL^3 \cdot L \right\} + \\
 &\quad + \frac{\alpha}{EI} \left\{ \frac{L^3}{3} + L^2 \cdot 2L \right\} + \frac{\alpha \Delta T}{h} \cdot L^2 = \\
 &= \frac{qL^4}{EI} \left[\frac{1}{8} + \frac{1}{2} + \frac{1}{2} + \frac{3}{2} \right] + \frac{\alpha L^3}{EI} \left[\frac{1}{3} + 2 \right] + \frac{\alpha \Delta T}{h} \cdot L^2 = \\
 &= \frac{21}{8} \frac{qL^4}{EI} + \frac{7}{3} \frac{\alpha L^3}{EI} + \frac{\alpha \Delta T}{h} \cdot L^2
 \end{aligned}$$

→ $L_{ve} = L_{vi}$ fornisce:

$$-\varepsilon_A X - \varepsilon_D L \left[\frac{3}{2} q L^2 + \frac{1}{L} X \right] =$$

$$= \frac{21}{8} \frac{q L^4}{EI} + \frac{7}{3} \frac{X L^3}{EI} + \frac{d\Delta \bar{T}}{h} L^2$$

$$X \left[\frac{7}{3} \frac{L^3}{EI} + \varepsilon_A + \varepsilon_D L^2 \right] = -\varepsilon_D \frac{3}{2} q L^3 - \frac{21}{8} \frac{q L^4}{EI} - \frac{d\Delta \bar{T}}{h} L^2$$

$\frac{5L^3}{3EI}$ $\frac{2L}{EI}$ $\frac{2L}{EI}$ $\frac{3}{8} \frac{q L^2}{EI}$

$$X \frac{L^3}{EI} \left[\frac{7}{3} + \frac{5}{3} + 2 \right] = - \frac{q L^4}{EI} \left[3 + \frac{21}{8} + \frac{3}{8} \right]$$

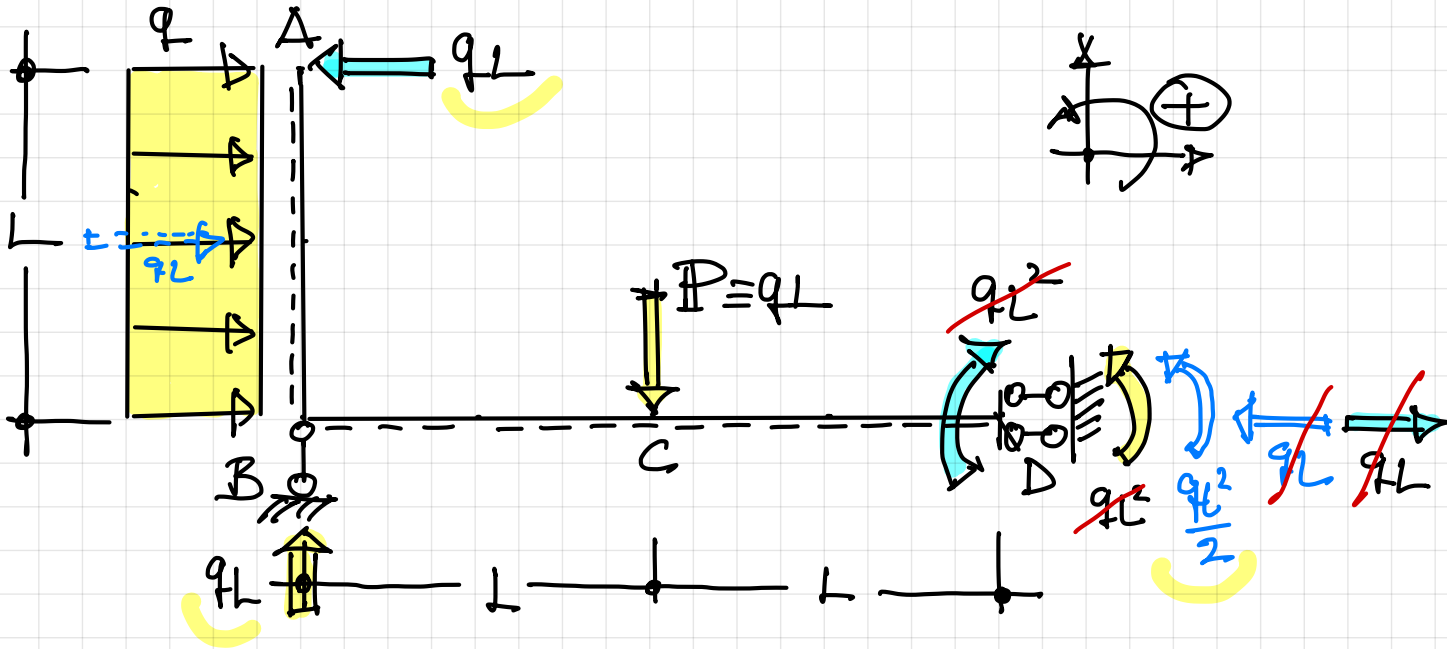
$\underbrace{\hspace{10em}}_6$ $\underbrace{\hspace{10em}}_6$

da cui:

$X = -qL$

NEGATIVA!
 VERSO OPPOSTO A QUELLO
 IPOTIZZATO

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



TRATTO AB $0 \leq z \leq L$

Free-body diagram of segment AB: A horizontal member of length z, fixed at A, with a horizontal distributed load q acting upwards. The reaction at A is a horizontal force qL acting to the left and a vertical force qL acting upwards. The internal forces at the free end are a horizontal force qL acting to the right and a vertical force qL acting downwards.

$$M^{(r)}(z) = q \frac{z^2}{2} - qL \cdot z \quad \left| \begin{array}{l} M_A = 0 \\ M_B = -\frac{qL^2}{2} \end{array} \right.$$

TRATTO BC $0 \leq z \leq L$

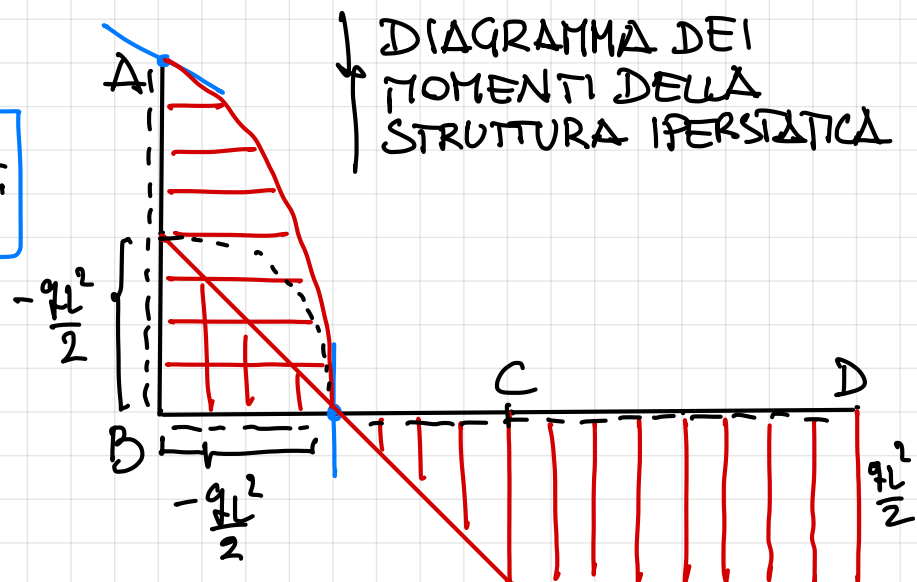
Free-body diagram of segment BC: A horizontal member of length z, fixed at B, with a vertical point load qL acting downwards at its midpoint. The reaction at B is a horizontal force qL acting to the left and a vertical force qL acting upwards. The internal forces at the free end are a horizontal force qL acting to the right and a vertical force qL acting downwards.

$$M^{(r)}(z) = qL \cdot z - \frac{qL^2}{2} \quad \left| \begin{array}{l} M_B = -\frac{qL^2}{2} \\ M_C = \frac{qL^2}{2} \end{array} \right.$$

TRATTO CD $L \leq z \leq 2L$

Free-body diagram of segment CD: A horizontal member of length z, fixed at C, with a horizontal distributed load q acting upwards. The reaction at C is a horizontal force qL acting to the left and a vertical force qL acting upwards. The internal forces at the free end are a horizontal force qL acting to the right and a vertical force qL acting downwards.

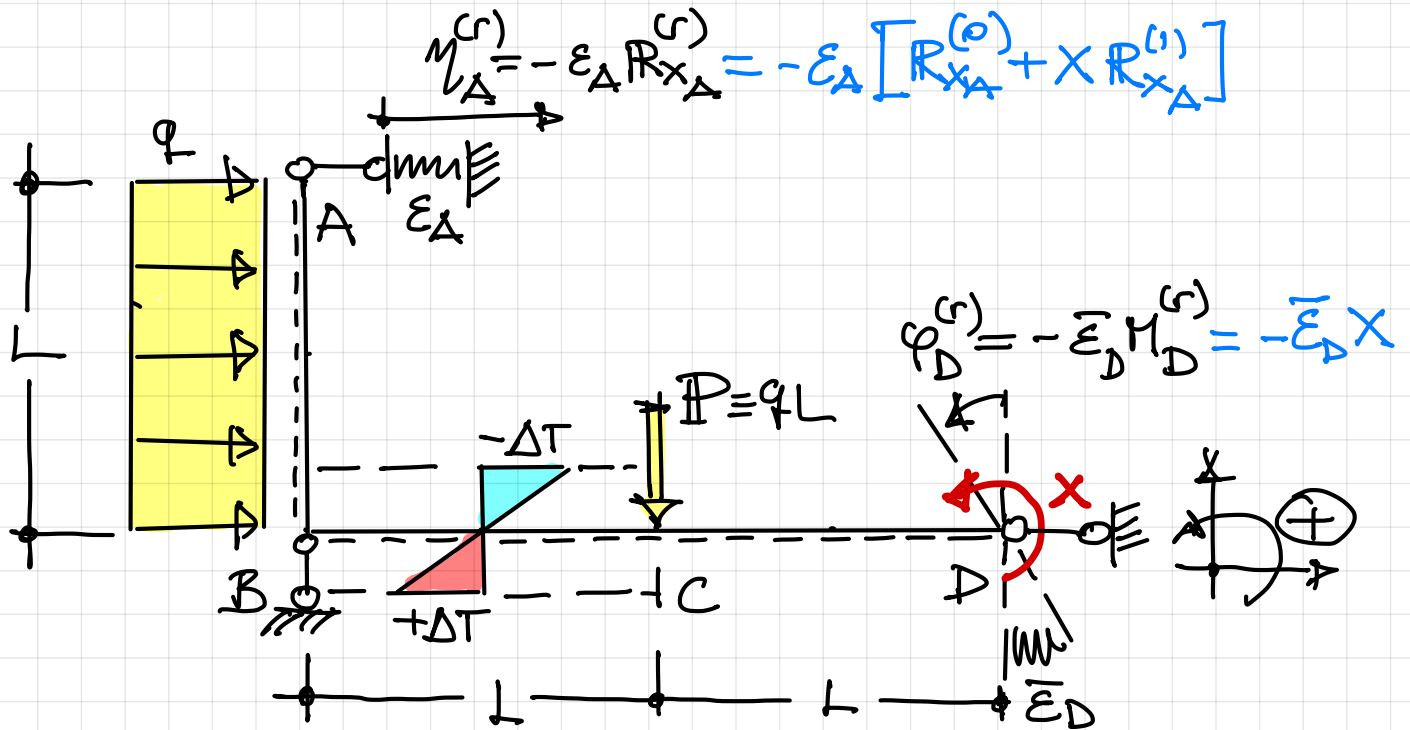
$$M^{(r)}(z) = \frac{qL^2}{2} \text{ cost.}$$



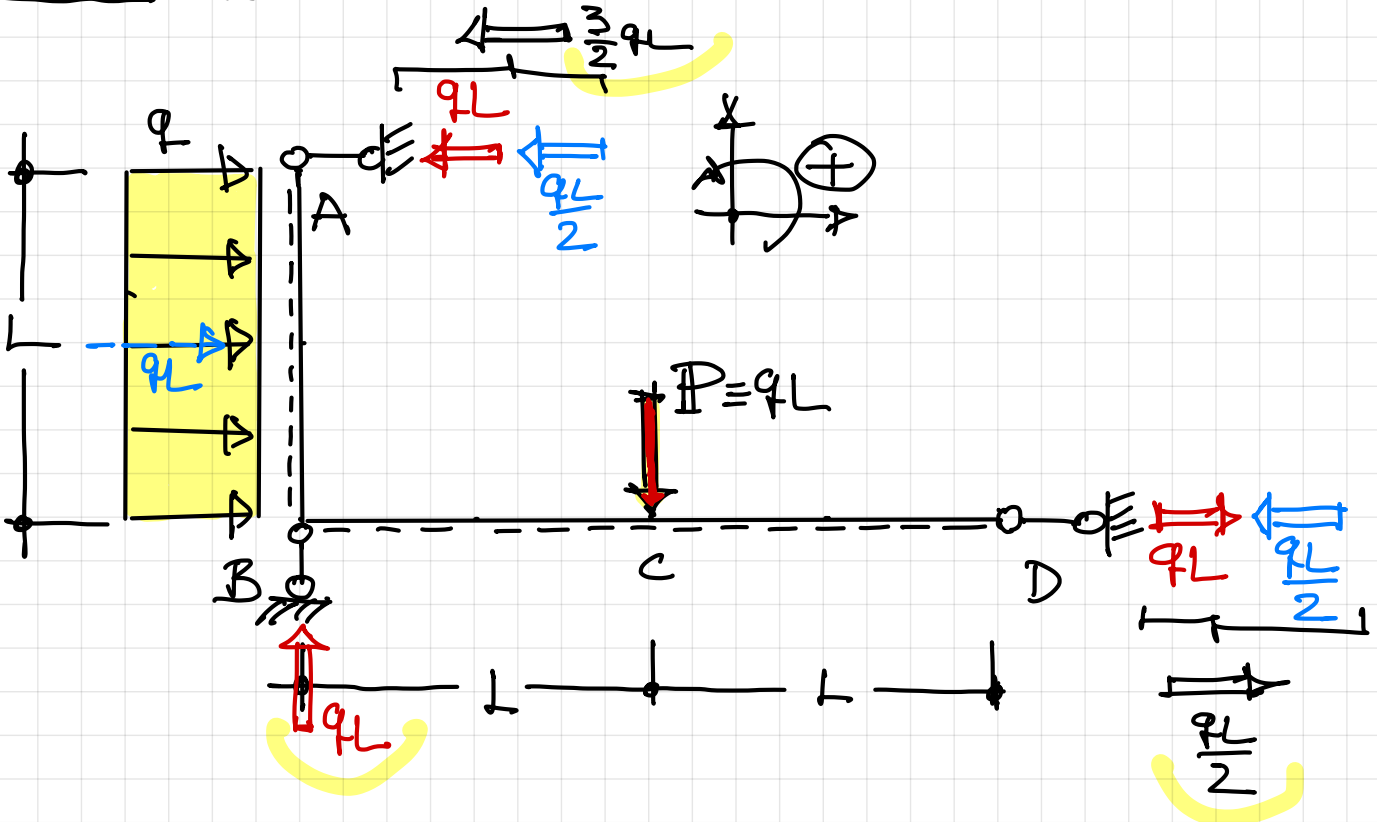
SOLUZIONE #2

VII

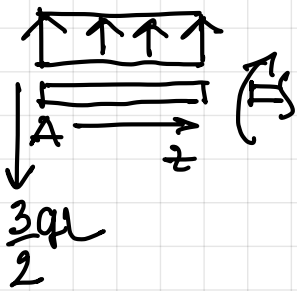
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI

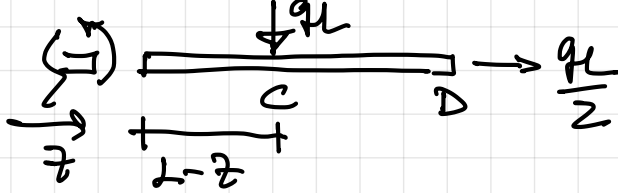


TRATTO AB $0 \leq z \leq L$



$$M^{(0)}(z) = q \frac{z^2}{2} - \frac{3}{2} qL \cdot z \quad \left\{ \begin{array}{l} M_A = 0 \\ M_B = -qL^2 \end{array} \right.$$

TRATTO BC $0 \leq z \leq L$

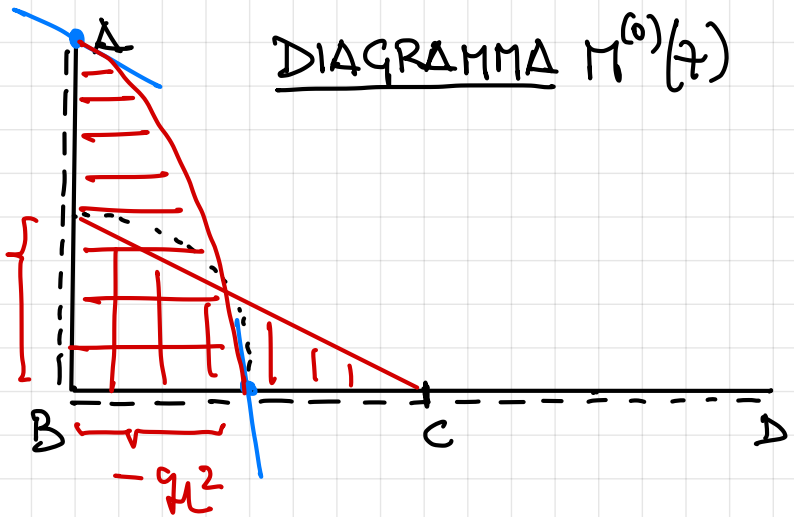


$$M^{(0)}(z) = -qL(L-z) \quad \left\{ \begin{array}{l} M_B = -qL^2 \\ M_C = 0 \end{array} \right.$$

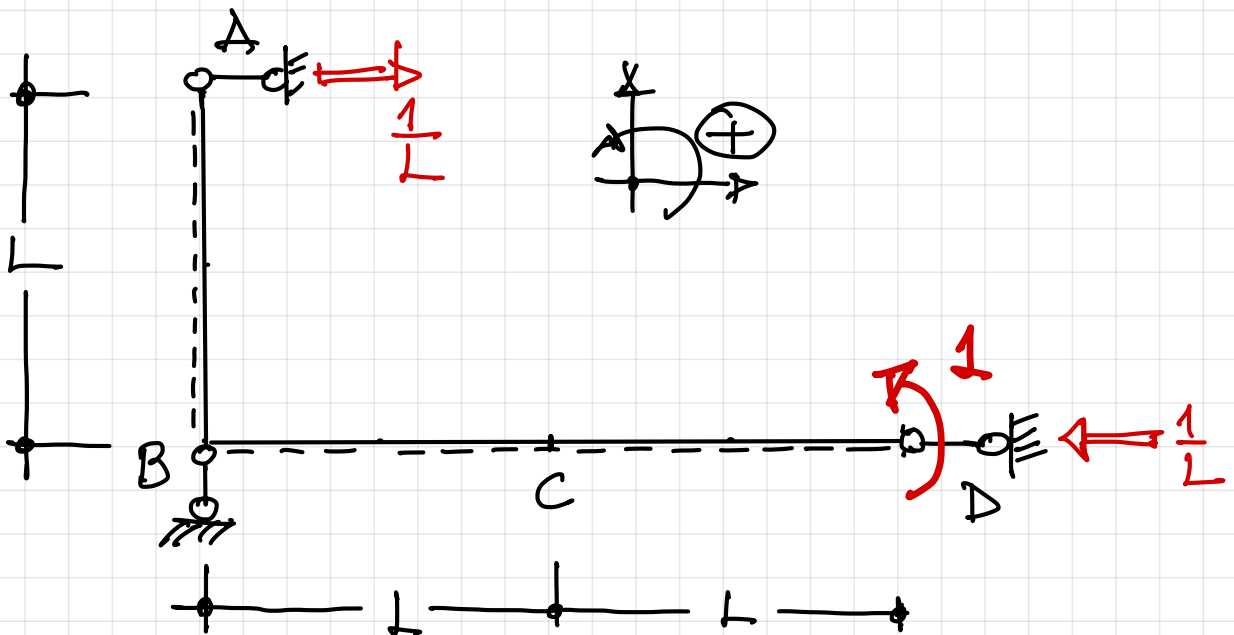
TRATTO CD

SCARICO

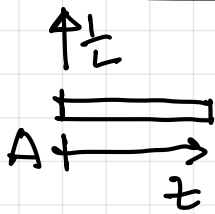
$$M^{(0)}(z) = 0$$



➡ SCHEMA [1] solo $x=1$



TRATTO AB $0 \leq z \leq L$

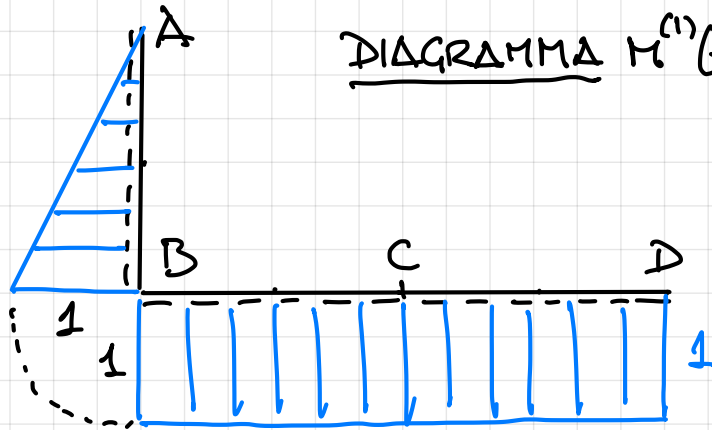


$$M^{(1)}(z) = \frac{z}{L} \quad \left\{ \begin{array}{l} M_A = 0 \\ M_B = 1 \end{array} \right.$$

TRATTO BD $0 \leq z \leq 2L$

IX

$$M^{(1)}(z) = 1 \quad \text{cost.}$$



$$\begin{aligned} L_{ve} &= \sum_i x_i \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = -\bar{\varepsilon}_D x + R_{x_A}^{(1)} (-\varepsilon_A) \left[R_{x_A}^{(0)} + x R_{x_A}^{(1)} \right] = \\ &= -\bar{\varepsilon}_D x - \frac{\varepsilon_A}{L} \left[-\frac{3}{2} qL + \frac{x}{L} \right] \end{aligned}$$

$$\begin{aligned} L_{vi} &= \int_{SK} \frac{M^{(1)} M^{(r)}}{EI} dSK + \int_{SK} \frac{M^{(1)} \alpha \Delta \bar{\Gamma}}{h} dSK = \\ &= \frac{1}{EI} \int_{SK} M^{(1)} M^{(0)} dSK + \frac{x}{EI} \int_{SK} [M^{(1)}]^2 dSK + \frac{\alpha \Delta \bar{\Gamma}}{h} \int_{SK} M^{(1)} dSK = \\ &= \frac{1}{EI} \left\{ \int_0^L \frac{z}{L} \left[\frac{qz^2}{2} - \frac{3}{2} qL \cdot z \right] dz + \int_0^L 1 \cdot \left[-qL(L-z) \right] dz \right\} + \\ &\quad + \frac{x}{EI} \left\{ \int_0^L \frac{z^2}{L^2} dz + \int_0^{2L} dz \right\} + \frac{\alpha \Delta \bar{\Gamma}}{h} \int_0^L 1 \cdot dz = \end{aligned}$$

X

$$\begin{aligned}
 &= \frac{1}{EI} \left\{ \frac{q}{2L} \left[\frac{z^4}{4} \right]_0^L - \frac{3}{2} q \left[\frac{z^3}{3} \right]_0^L - qL^2 \left[z \right]_0^L + qL \left[\frac{z^2}{2} \right]_0^L \right\} + \\
 &+ \frac{X}{EI} \left\{ \frac{1}{L^2} \left[\frac{z^3}{3} \right]_0^L + \left[z \right]_0^L \right\} + \frac{\alpha \Delta T}{h} \cdot \left[z \right]_0^L = \\
 &= \frac{1}{EI} \left\{ \frac{q}{2L} \cdot \frac{L^4}{4} - \frac{3}{2} q \frac{L^3}{3} - qL^2 \cdot L + \frac{qL}{2} \cdot L^2 \right\} + \\
 &+ \frac{X}{EI} \left\{ \frac{1}{3L^2} \cdot L^3 + 2L \right\} + \frac{\alpha \Delta T}{h} \cdot L = \\
 &= \frac{qL^3}{EI} \left\{ \frac{1}{8} - \frac{1}{2} - 1 + \frac{1}{2} \right\} + \frac{XL}{EI} \left[\frac{1}{3} + 2 \right] + \frac{\alpha \Delta T}{h} \cdot L = \\
 &= -\frac{7}{8} \frac{qL^3}{EI} + \frac{7}{3} \frac{XL}{EI} + \frac{\alpha \Delta T}{h} L
 \end{aligned}$$



$L_{ve} = L_{vi}$ forliscce:

$$\begin{aligned}
 -\bar{\varepsilon}_D X - \frac{\varepsilon_A}{L} \left[-\frac{3}{2} qL + \frac{X}{L} \right] &= -\frac{7}{8} \frac{qL^3}{EI} + \frac{7}{3} \frac{XL}{EI} + \frac{\alpha \Delta T}{h} L \\
 X \left[\frac{7}{3} \frac{L}{EI} + \bar{\varepsilon}_D + \frac{\varepsilon_A}{L^2} \right] &= +\frac{7}{8} \frac{qL^3}{EI} - \frac{\alpha \Delta T}{h} L + \varepsilon_A \frac{3}{2} q
 \end{aligned}$$

$\frac{7}{3} \frac{L}{EI}$ (blue)
 $\bar{\varepsilon}_D$ (blue)
 $\frac{\varepsilon_A}{L^2}$ (blue)
 $+\frac{7}{8} \frac{qL^3}{EI}$ (blue)
 $-\frac{\alpha \Delta T}{h} L$ (blue)
 $+\varepsilon_A \frac{3}{2} q$ (blue)

$$X \frac{1}{EI} \left[\frac{7}{3} + 2 + \frac{5}{3} \right] = \frac{qL^2}{EI} \left\{ \frac{7}{8} - \frac{3}{8} + \frac{5}{2} \right\}$$

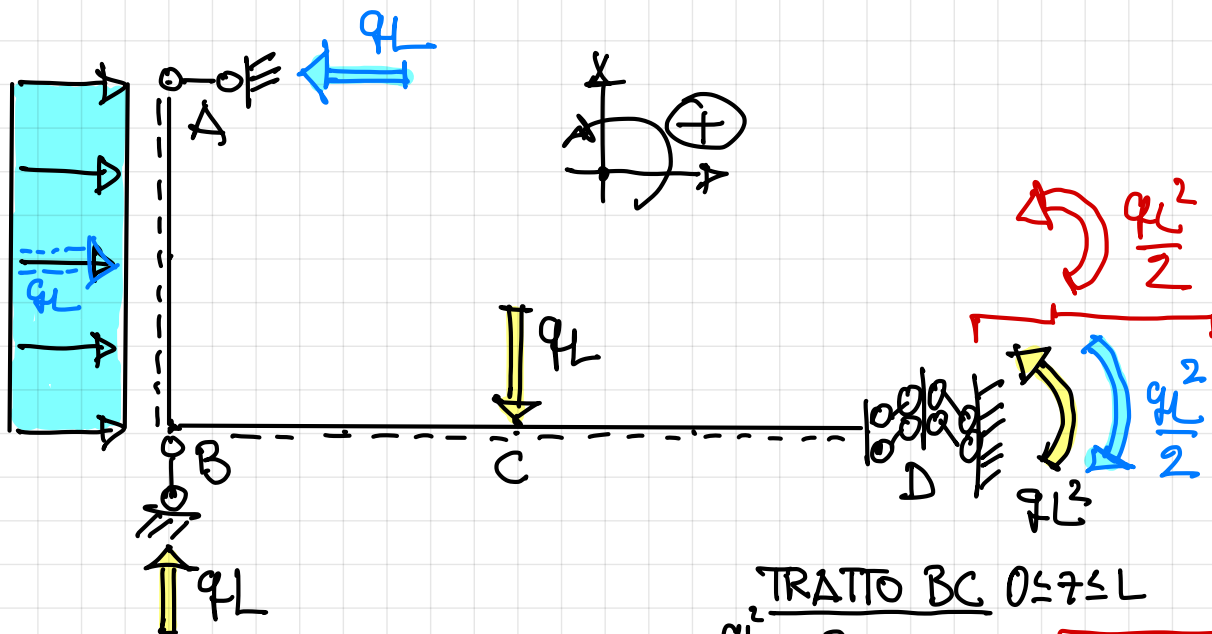
$\frac{7}{3} + 2 + \frac{5}{3}$ (green 6)
 $\frac{7}{8} - \frac{3}{8} + \frac{5}{2}$ (green 3)

$6X = 3qL^2$ da cui $X = \frac{qL^2}{2}$

OK!
 cri. RV
 di pag. VI

Diagram of a frame structure ABCD. The structure consists of a vertical column AB of height L and a horizontal beam BC of length $2L$. A uniformly distributed load q acts to the right on the beam. The beam is supported by a pin at B and a roller at D. The column is fixed at A. A horizontal force $P = qL$ acts downwards at the midpoint C of the beam. The beam has a temperature gradient with $+\Delta T$ on the bottom and $-\Delta T$ on the top. The column has a horizontal displacement ϵ_A at A. The diagram includes internal force diagrams: a linear moment diagram on the beam with a maximum value of $PL/2$ at C, and a linear shear force diagram on the column with a maximum value of qL at A. The diagram also shows the equivalent nodal loads at A: a horizontal force $R_A^{(0)}$ and a moment $R_A^{(1)}$. The diagram is labeled with various parameters: q , L , $P = qL$, ΔT , ϵ_A , $R_A^{(0)}$, $R_A^{(1)}$, and the nodal load equations: $M_A^{(0)} = -\epsilon_A R_A^{(1)} = -\epsilon_A [R_A^{(0)} + X R_A^{(1)}]$ and $\phi_D^{(0)} = -\epsilon_D M_D^{(0)} = -\epsilon_D [P + X P]$.

$$\varphi_D^{(5)} = -\mathbb{E}_D M_D^{(5)} = -\mathbb{E}_D [M_D^{(0)} + x M_D^{(1)}]$$



TRATTO BC $0 \leq z \leq L$

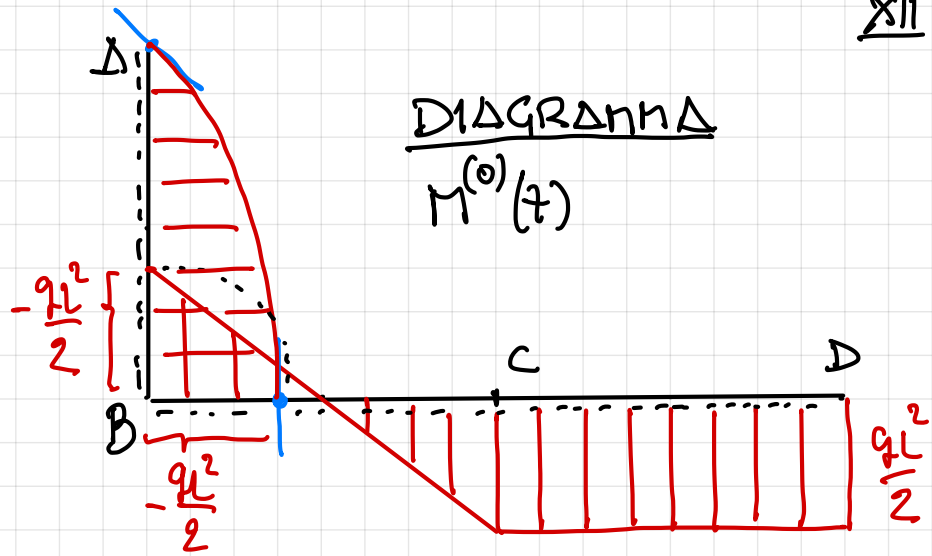
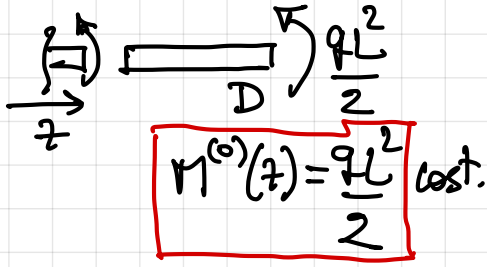
$$r^{(0)}_i = qz^2 - qLz$$

$$\begin{aligned} \uparrow M_A &= 0 \\ \uparrow M_B &= -\frac{qL^2}{2} \end{aligned}$$

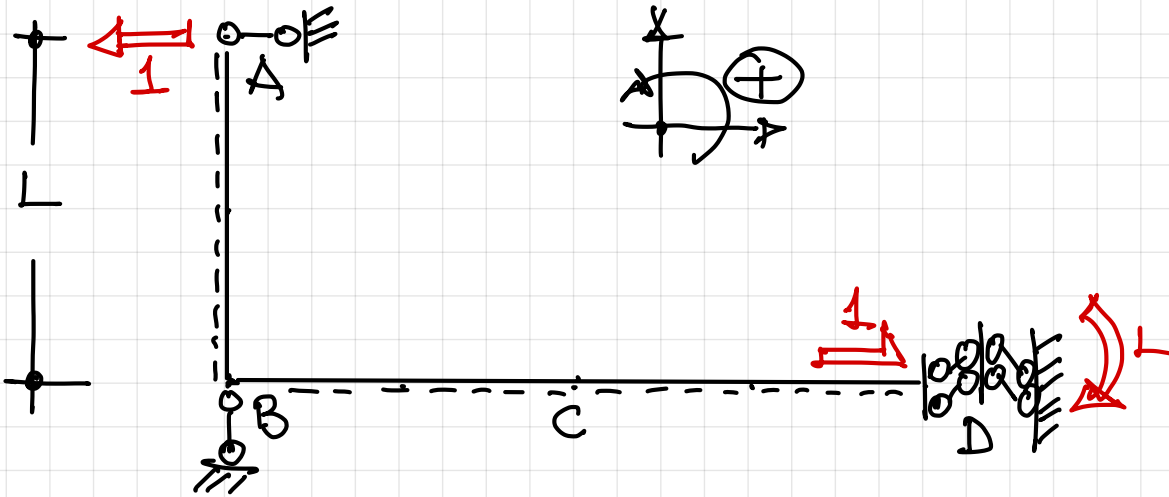
$$M^{(0)}(t) = q_L \cdot t - \frac{q_L^2}{2}$$

$$M_B = -\frac{qL^2}{2}; M_C = \frac{qL^2}{2}$$

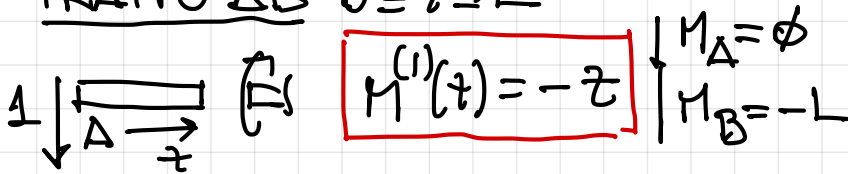
TRATTO CD $L \leq z \leq 2L$



SCHEMA [1] solo $x=1$



TRATTO AB $0 \leq z \leq L$



TRATTO BD $0 \leq z \leq 2L$

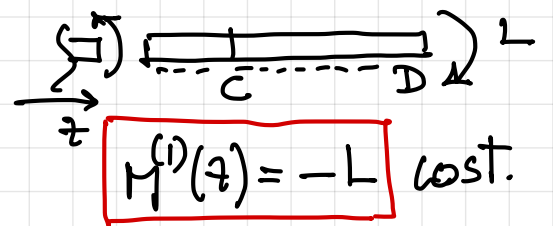
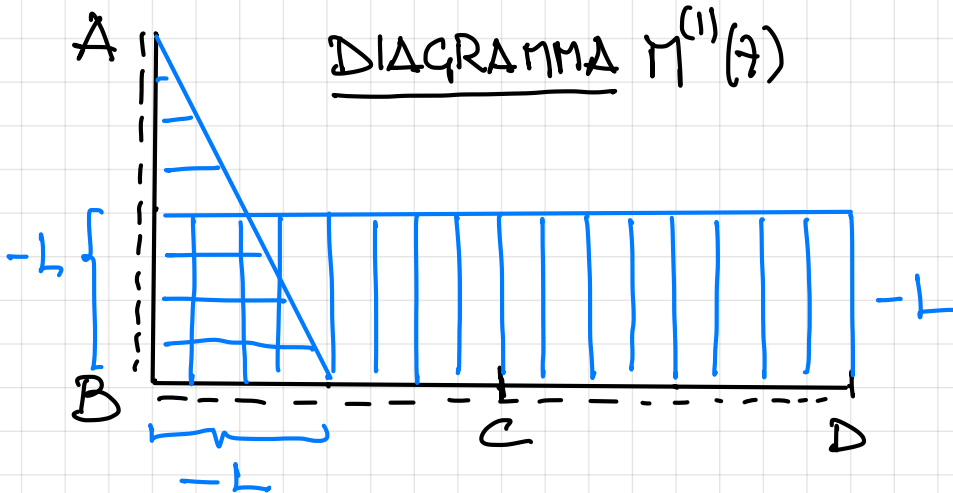


DIAGRAMMA $M^{(1)}(z)$



$$\Rightarrow \underline{L_{ve}} = \sum_i X_i \underbrace{\eta_i^{(r)}}_{=\phi} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$= \underbrace{R_{x_A}^{(1)}}_{-1} (-\varepsilon_A) \left[\underbrace{R_{x_A}^{(0)}}_{-qL} + X \underbrace{R_{x_A}^{(1)}}_{-1} \right] + \underbrace{M_D^{(1)}}_{-L} (-\bar{\varepsilon}_D) \left[\underbrace{M_D^{(0)}}_{\frac{qL^2}{2}} + X \underbrace{M_D^{(1)}}_{-1} \right] =$$

$$= -\varepsilon_A [qL + X] + \bar{\varepsilon}_D L \left[\frac{qL^2}{2} - XL \right]$$

$$\Rightarrow \underline{L_{vi}} = \int_{str} M^{(1)} \frac{\eta^{(r)}}{EI} dstr + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} dstr =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} dstr + \frac{X}{EI} \int_{str} [M^{(1)}]^2 dstr + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} dstr =$$

$$= \frac{1}{EI} \left\{ \int_0^L (-z) \left[\frac{qz^2}{2} - qLz \right] dz + \int_0^L -L \left[qLz - \frac{qL^2}{2} \right] dz + \int_L^{2L} -L \left[\frac{qL^2}{2} \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L z^2 dz + \int_0^{2L} L^2 dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L -L dz =$$

$$= \frac{1}{EI} \left[-\frac{q}{2} \left[\frac{z^4}{4} \right]_0^L + qL \left[\frac{z^3}{3} \right]_0^L - qL^2 \left[\frac{z^2}{2} \right]_0^L + \frac{qL^3}{2} [z]_0^L - \frac{qL^3}{2} [z]_L^{2L} + \right.$$

$$\left. + \frac{X}{EI} \left[\left[\frac{z^3}{3} \right]_0^L + L^2 [z]_0^{2L} \right] - \frac{\alpha \Delta T}{h} L [z]_0^L = \right.$$

$$= \frac{1}{EI} \left[-\frac{q}{8} \cdot L^4 + \frac{qL}{3} \cdot L^3 - \frac{qL^2}{2} \cdot L^2 + \frac{qL^3}{2} \cdot L - \frac{qL^3}{2} \cdot L \right] +$$

$$+ \frac{X}{EI} \left[\frac{L^3}{3} + L^2 \cdot 2L \right] - \frac{\alpha \Delta T}{h} \cdot L \cdot L =$$

$$= \frac{qL^4}{EI} \left[-\frac{1}{8} + \frac{1}{3} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \right] + \frac{XL^3}{EI} \left[\frac{1}{3} + 2 \right] - \alpha \frac{\Delta T}{h} \cdot L^2 =$$

$$= \underbrace{-\frac{qL^4}{EI} \frac{7}{24} + \frac{7}{3} \frac{XL^3}{EI} - \alpha \frac{\Delta T}{h} L^2}_{=0}$$

⇒ $L_{ve} = L_{vi}$ formisco

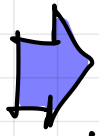
$$-E_A [qL + X] + \bar{E}_D L \left[\frac{qL^2}{2} - XL \right] =$$

$$= -\frac{qL^4}{EI} \frac{7}{24} + \frac{7}{3} \frac{XL^3}{EI} - \alpha \frac{\Delta T}{h} L^2$$

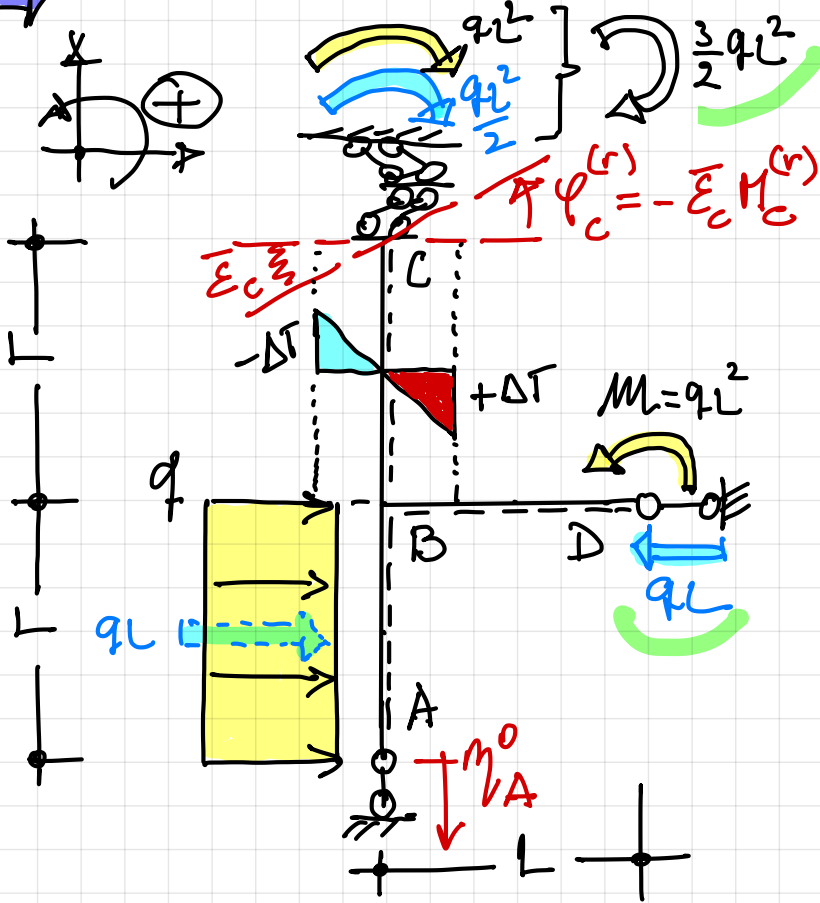
$$X \left[\frac{7}{3} \frac{L^3}{EI} + \underbrace{E_A}_{\frac{5L^3}{3EI}} + \underbrace{\bar{E}_D}_{\frac{2L}{EI}} L^2 \right] = \frac{qL^4}{EI} \cdot \frac{7}{24} + \underbrace{\alpha \frac{\Delta T}{h}}_{\frac{3qL^2}{8EI}} L^2 - \underbrace{E_A}_{\frac{5L^3}{3EI}} qL + \underbrace{\bar{E}_D}_{\frac{2L}{EI}} \frac{qL^3}{2}$$

$$X \frac{L^3}{EI} \left[\frac{7}{3} + \frac{5}{3} + 2 \right] = \frac{qL^4}{EI} \left[\frac{7}{24} + \frac{3}{8} - \frac{5}{3} + 1 \right] = 0$$

da cui $X=0$ OK! cf. RV di p. VI



STRUTTURA REALE



TRATTO AB $0 \leq z \leq L$

$$\begin{aligned} & \left[\begin{array}{c} \text{Diagram of beam AB with distributed load } q \end{array} \right] \Rightarrow \boxed{M(z) = -\frac{qz^2}{2}} \\ & \left\{ \begin{array}{l} M_A = 0 \\ M_B = -\frac{qL^2}{2} \end{array} \right. \end{aligned}$$

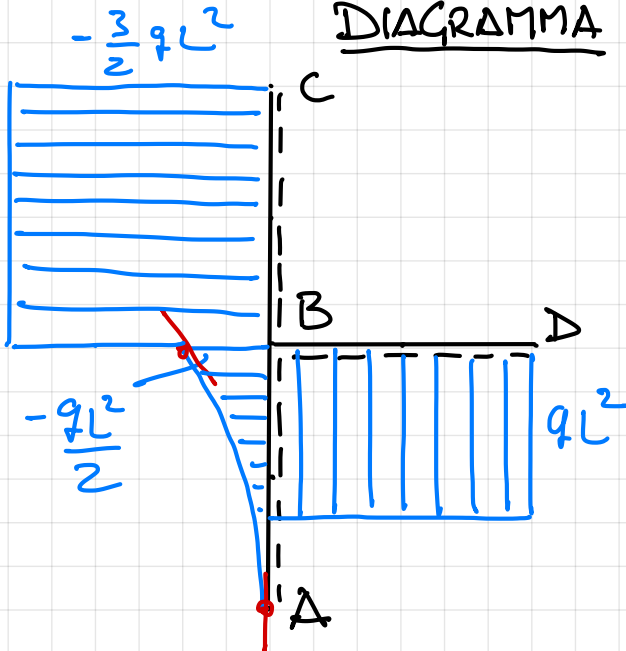
TRATTO BC $L \leq z \leq 2L$

$$\begin{aligned} & \left[\begin{array}{c} \text{Diagram of beam BC with distributed load } q \end{array} \right] \Rightarrow \boxed{M(z) = -\frac{3qL^2}{2} \text{ cost.}} \end{aligned}$$

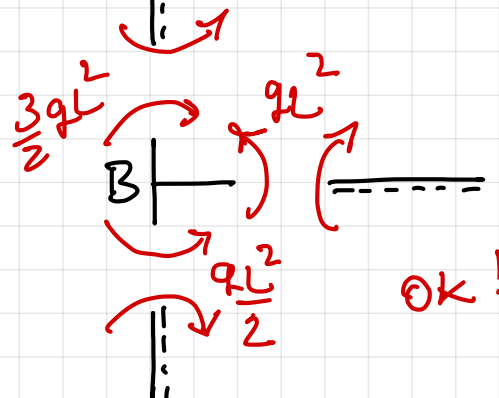
TRATTO BD $0 \leq z \leq L$

$$\begin{aligned} & \left[\begin{array}{c} \text{Diagram of beam BD with point load } qL \end{array} \right] \Rightarrow \boxed{M(z) = qL^2 \text{ cost.}} \end{aligned}$$

DIAGRAMMA $M(z)$

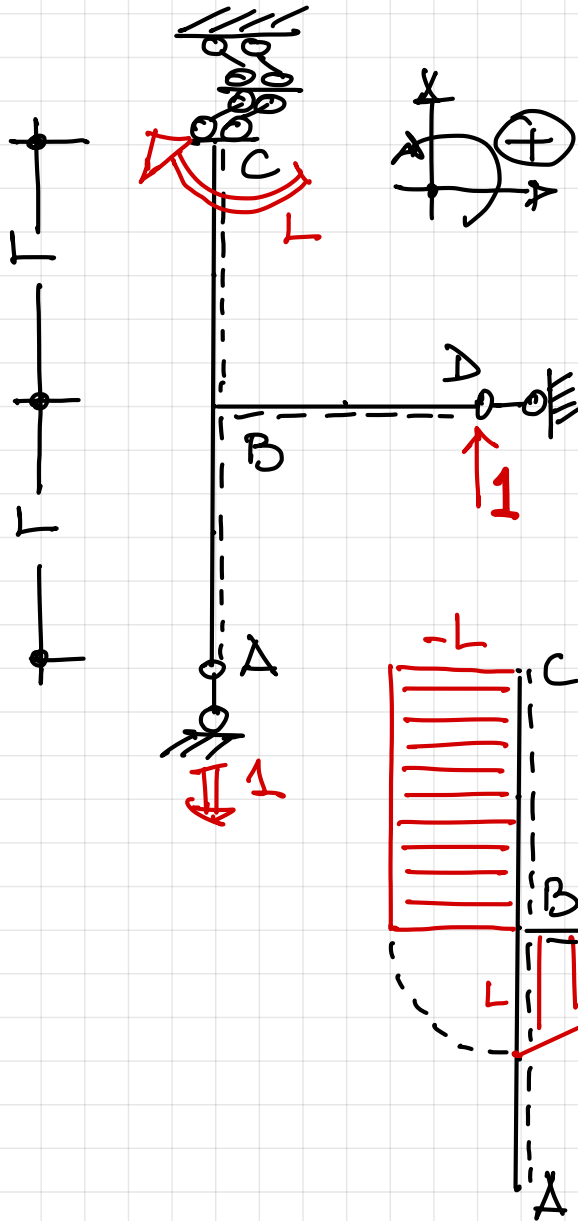


VERIFICA AL NODO TRIPLO B





STRUTTURA FITTIZIA PER IL CALCOLO DELLO SPOSTAMENTO VERTICALE DELLA SEZIONE D



TRATTO AB $0 \leq z \leq L$

SCARICO $M^{(f)}(z) = \phi$

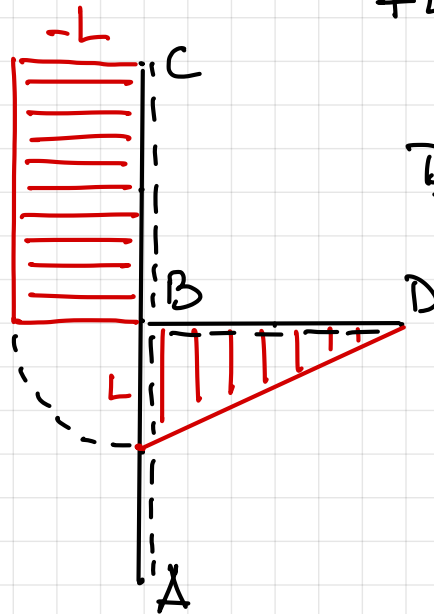
TRATTO BC $L \leq z \leq 2L$

$M^{(f)}(z) = -L \cos t$

TRATTO BD $0 \leq z \leq L$

$M^{(f)}(z) = L - z$

DIAGRAMMA $M^{(f)}(z)$



$$\begin{aligned}
 Lve &= 1 \cdot \eta_D + \sum_j R_j^{(f)} \eta_j^{(r)} = \\
 &= 1 \cdot \eta_D + \underbrace{R_{yA}^{(f)}}_{-1} (-\eta_A^0) + \underbrace{M_C^{(f)}}_{-L} \varphi_C^{(r)} = \\
 &= \eta_D + \eta_A^0 - \bar{E}_c \frac{3}{2} q L^3
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \underline{L_{vi}} &= \int_{str} M^{(f)} \frac{1}{EI} dstr + \int_{str} M^{(f)} \frac{\alpha \bar{\Delta T}}{h} dstr = \\
 &= \frac{1}{EI} \left\{ \int_L^{2L} (-L) \left(-\frac{3}{2} q L^2 \right) dz + \int_0^L (L-z) q L^2 dz \right\} + \frac{\alpha \bar{\Delta T}}{h} \int_L^{2L} \left(-\frac{L}{2} \right) dz = \\
 &= \frac{1}{EI} \left\{ \frac{3}{2} q L^3 [z]_L^{2L} + q L^3 [z]_0^L - q L^2 \left[\frac{z^2}{2} \right]_0^L \right\} - \frac{\alpha \bar{\Delta T}}{h} L [z]_L^{2L} = \\
 &= \frac{1}{EI} \left\{ \frac{3}{2} q L^4 + q L^4 - \frac{q L^4}{2} \right\} - \frac{\alpha \bar{\Delta T}}{h} L^2 = \\
 &= \underline{\underline{\frac{2 q L^4}{EI} - \frac{\alpha \bar{\Delta T}}{h} L^2}}
 \end{aligned}$$

$$\Rightarrow L_{ve} = L_{vi} \text{ fornisce}$$

$$\eta_D + \underbrace{\eta_A^0}_{\frac{q L^4}{2 EI}} - \underbrace{\bar{E}_c \frac{3}{2} q L^3}_{\frac{L}{EI}} = \cancel{\frac{2 q L^4}{EI}} - \cancel{\frac{\alpha \bar{\Delta T}}{h} L^2}_{\frac{2 q L^2}{EI}}$$

de cui

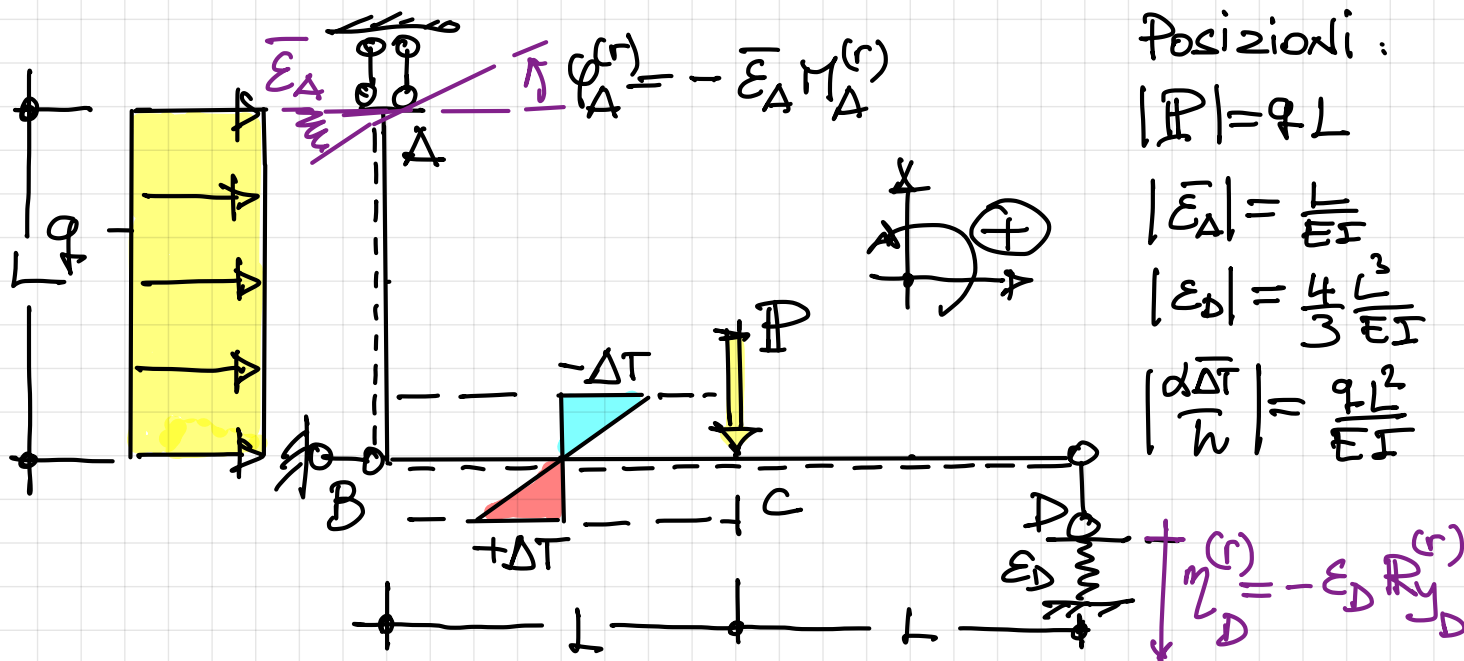
$$\eta_D = \frac{3}{2} \frac{q L^4}{EI} - \frac{q L^4}{2 EI} = \underline{\underline{\frac{q L^4}{EI}}} \quad \left| \begin{array}{l} \text{POSITIVO} \\ \text{VERSO L'ALTO} \end{array} \right.$$

MECCANICA delle STRUTTURE - P. FUSCHI

PROVA SCRITTA del 29 GENNAIO 2025

TIPO
2

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI



ES. #2 CALCOLARE LO SPOST. ORIZZONTALE DELLA SEZ. D DELLA STRUTTURA SEGUENTE CON IL METODO DELLA FORZA UNITARIA

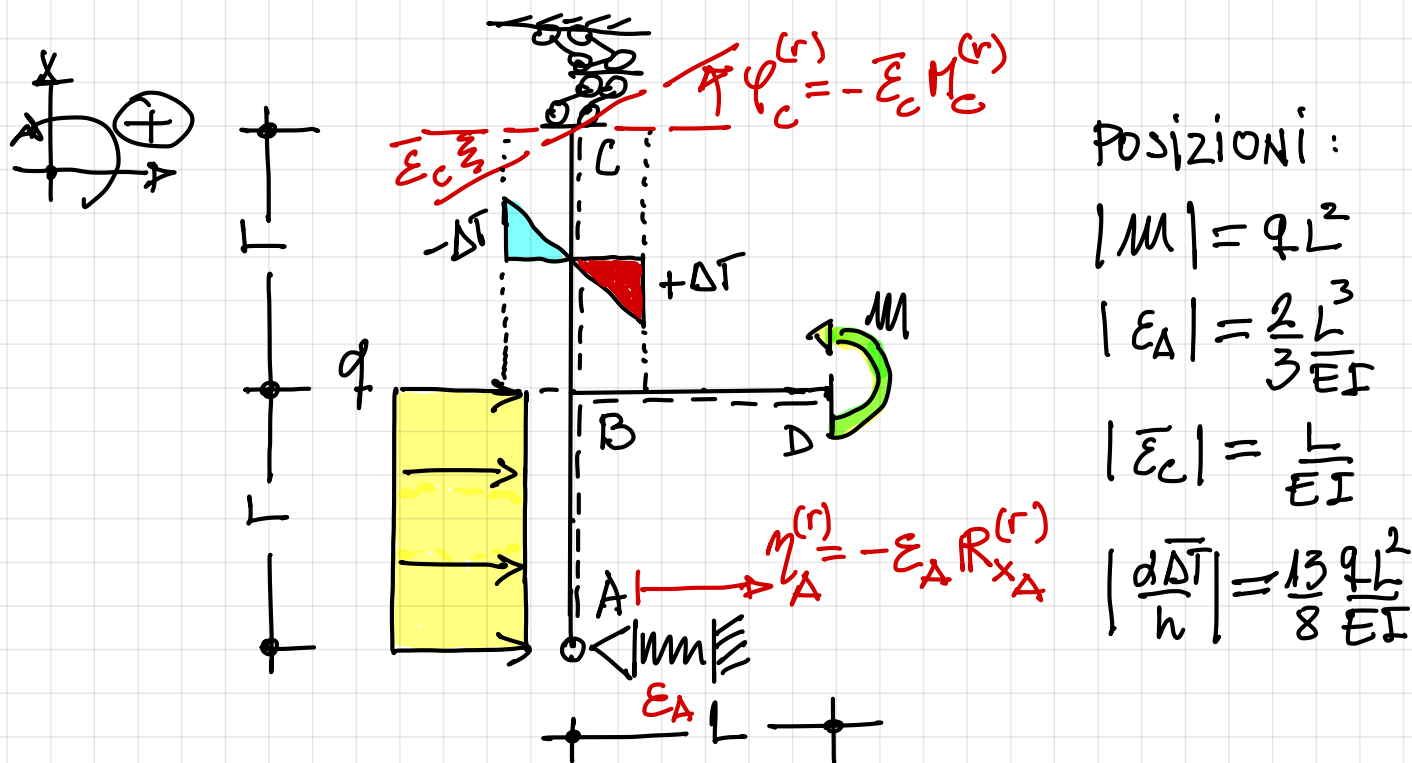


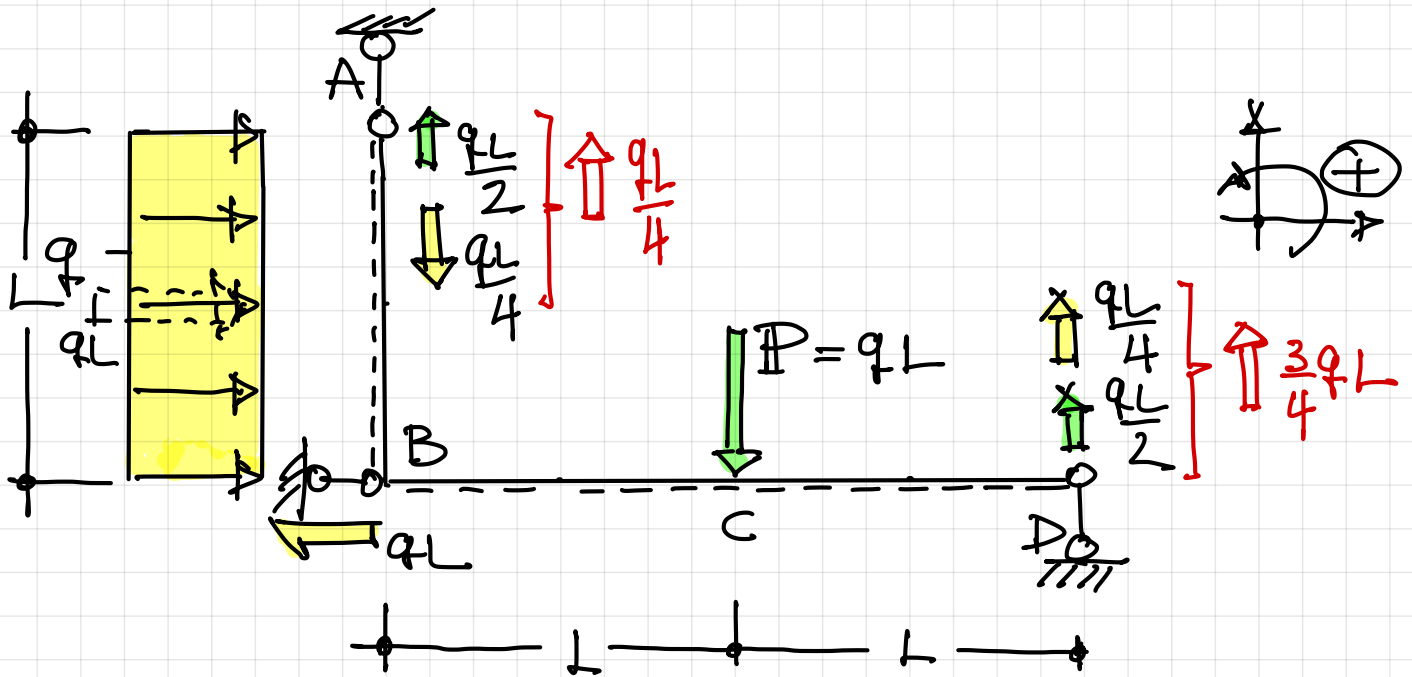
Diagram illustrating the structure and internal forces:

- Horizontal beam of length $2L$ with a hinge support at the left end (B) and a roller support at the right end (D).
- Uniformly distributed load q acts downwards over the entire length of the beam.
- Vertical column of height L is fixed at the bottom (A) and has a hinge support at the top (C).
- A horizontal force $P = qL$ acts to the right at the top of the column (C).
- The diagram shows the internal force distributions: a linear temperature profile across the beam with a maximum difference of ΔT , and a constant axial force $E_A \Delta$ across the column.
- The displacement at the top of the column is labeled Δ .

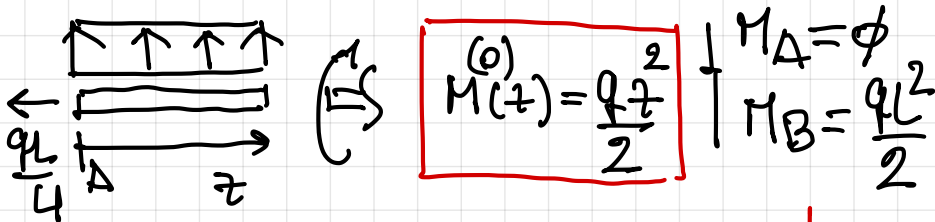
$$\epsilon = -\epsilon_D [R_{y_D}^{(0)} + X R_{y_D}^{(1)}]$$

$$Z_D^{(r)} = -\epsilon_D \bar{Y}_D^{(r)} = 0$$

SCHEMA [0] - SOLO CARICHI ESTERNI



TRATTO AB $0 \leq z \leq L$



TRATTO BC $0 \leq z \leq L$

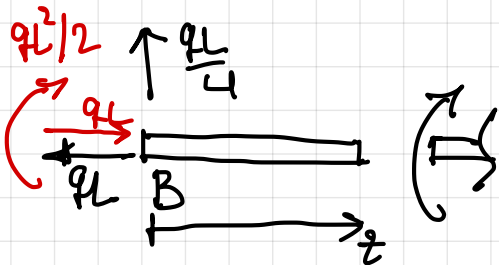
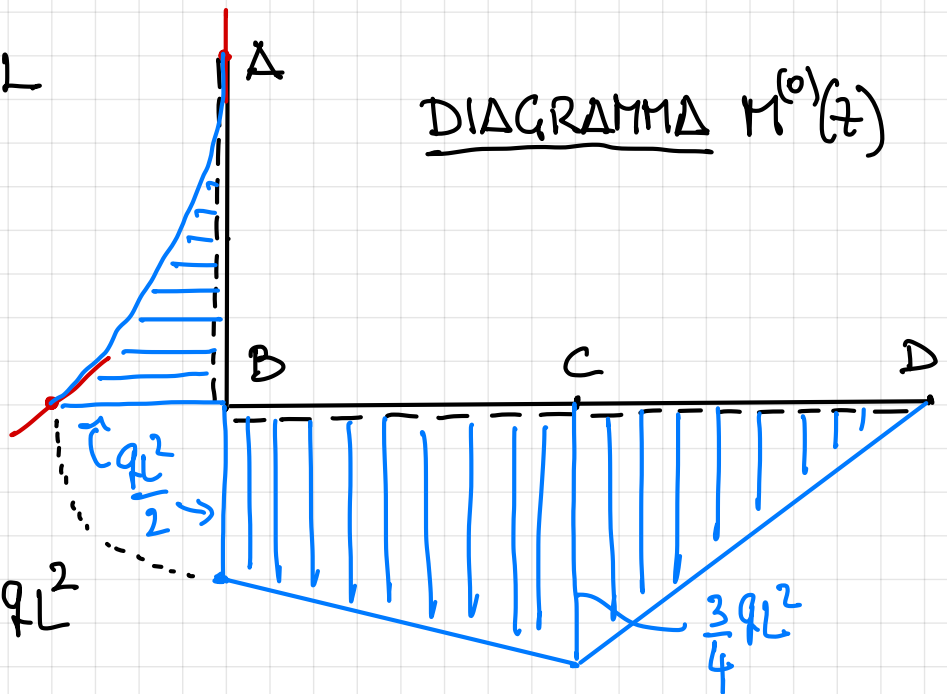


DIAGRAMMA $M^{(0)}(z)$



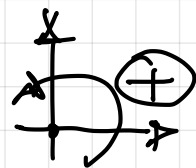
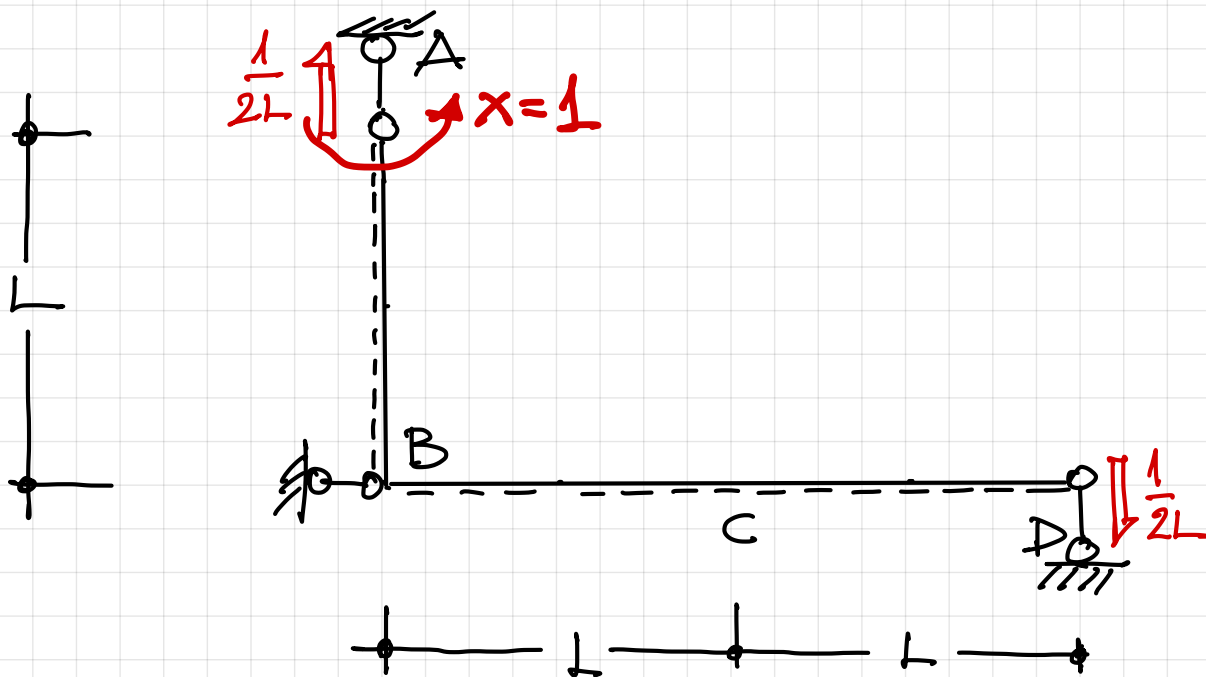
$$M^{(0)}(z) = \frac{qL^2}{2} + \frac{qL}{4}z$$

$$M_B = \frac{qL^2}{2}; M_C = \frac{3}{4}qL^2$$

TRATTO CD $L \leq z \leq 2L$

$$M^{(0)}(z) = \frac{3}{4}qL(2L - z) \quad \left\{ \begin{array}{l} M_C = \frac{3}{4}qL^2 \\ M_D = 0 \end{array} \right.$$

➔ SCHEMA [1] SOLO $X=1$



TRATTO AB $0 \leq z \leq L$

$\frac{1}{2L}$ \Rightarrow $M^{(1)}(z) = -1$ cost.

TRATTO BD $0 \leq z \leq 2L$

$\frac{1}{2L}$ \Rightarrow $M^{(1)}(z) = \frac{z}{2L} - 1$ $\left\{ \begin{array}{l} M_B = -1 \\ M_D = 0 \end{array} \right.$

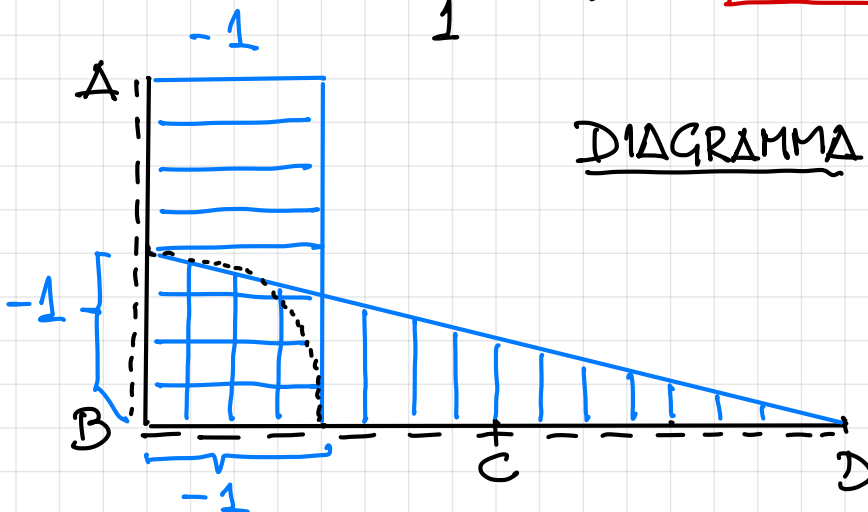
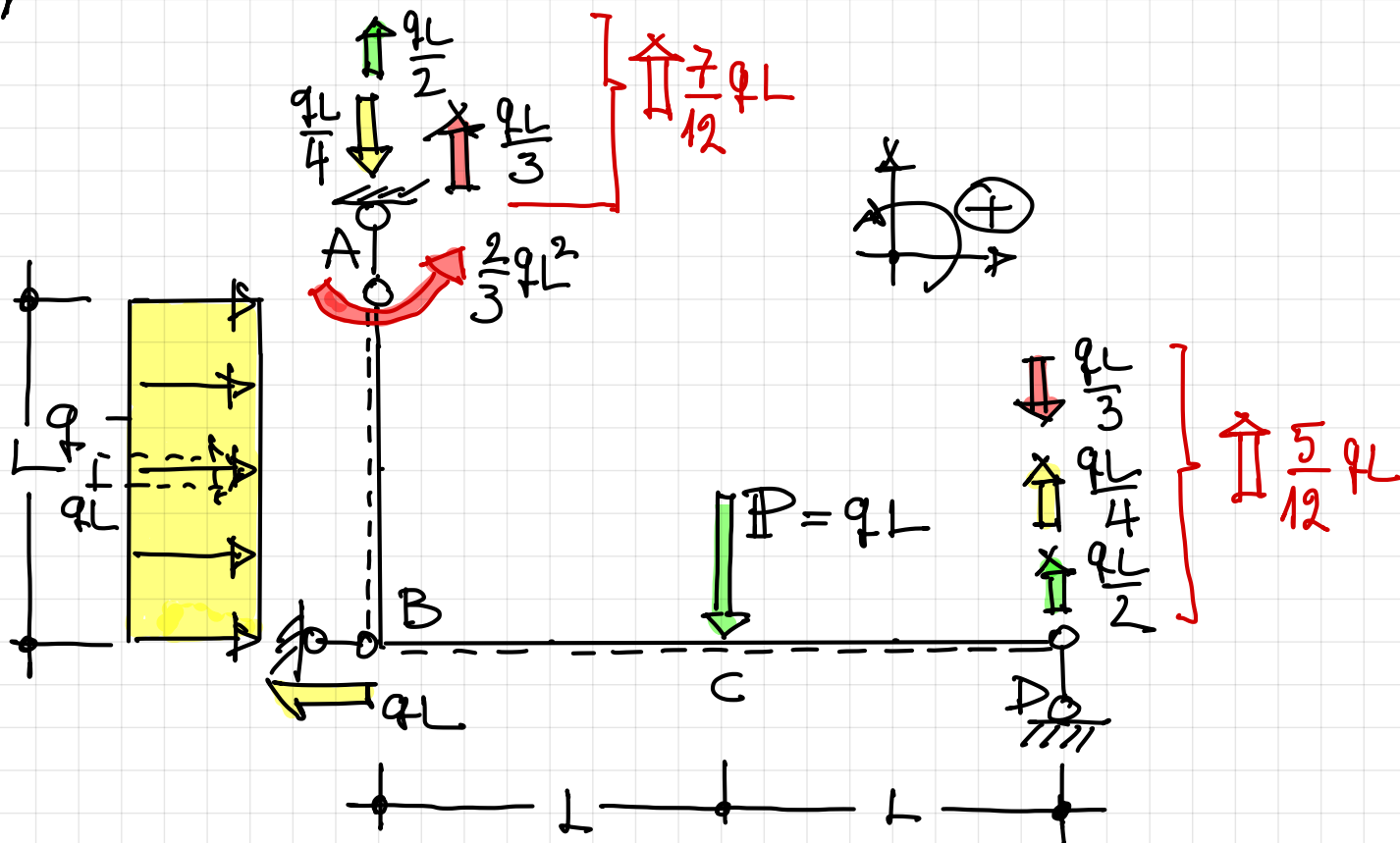


DIAGRAMMA $M^{(1)}(z)$

➔ $Lve = \sum_{i=1}^n x_i \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} =$
 $= -\bar{\varepsilon}_A X + \underbrace{R_{yD}^{(1)}}_{-\frac{1}{2L}} (-\varepsilon_D) \left[\underbrace{R_{yD}^{(0)}}_{\frac{3}{4}9L} + X \underbrace{R_{yD}^{(1)}}_{-\frac{1}{2L}} \right] =$
 $= -\bar{\varepsilon}_A X + \frac{\varepsilon_D}{2L} \left[\frac{3}{4}9L - \frac{X}{2L} \right]$

$$\begin{aligned}
\Rightarrow \underline{L_{vi}} &= \int_{str} M^{(1)} \frac{M^{(0)}}{EI} dstr + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} dstr = \\
&= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} dstr + \frac{\alpha}{EI} \int_{str} [M^{(1)}]^2 dstr + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} dstr = \\
&= \frac{1}{EI} \left\{ \int_0^L (-1) \frac{qz^2}{2} dstr + \int_0^L \left(\frac{z}{2L} - 1 \right) \left[\frac{qL^2}{2} + \frac{qL}{4} z \right] dstr + \right. \\
&\quad \left. \underbrace{\left(\frac{z}{2L} - 1 \right) \left(\frac{3}{2} qL^2 - \frac{3}{4} qLz \right)}_{\frac{3}{4} qLz - \frac{3}{2} qL^2 - \frac{3}{8} qz^2 + \frac{3}{4} qLz} + \int_L^{2L} \left(\frac{z}{2L} - 1 \right) \left[\frac{3}{4} qL(2L - z) \right] dstr \right\} + \\
&\quad + \frac{\alpha}{EI} \left\{ \int_0^L 1 dz + \int_0^{2L} \underbrace{\left(\frac{z}{2L} - 1 \right)^2}_{\frac{z^2}{4L^2} + 1 - \frac{z}{L}} dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L \left(\frac{z}{2L} - 1 \right) dstr = \\
&= \frac{1}{EI} \left[-\frac{q}{2} \left[\frac{z^3}{3} \right]_0^L + \frac{qL}{4} \left[\frac{z^2}{2} \right]_0^L - \frac{qL^2}{2} [z]_0^L + \frac{q}{8} \left[\frac{z^3}{3} \right]_0^L - \frac{qL}{4} \left[\frac{z^2}{2} \right]_0^L + \right. \\
&\quad \left. + \frac{3}{4} qL \left[\frac{z^2}{2} \right]_L^{2L} - \frac{3}{2} qL^2 [z]_L^{2L} - \frac{3}{8} q \left[\frac{z^3}{3} \right]_L^{2L} + \frac{3}{4} qL \left[\frac{z^2}{2} \right]_L^{2L} \right] + \\
&\quad + \frac{\alpha}{EI} \left[[z]_0^L + \frac{1}{4L^2} \left[\frac{z^3}{3} \right]_0^{2L} + [z]_0^{2L} - \frac{1}{L} \left[\frac{z^2}{2} \right]_0^{2L} \right] + \\
&\quad + \frac{\alpha \Delta T}{h} \left[\frac{1}{2L} \left[\frac{z^2}{2} \right]_0^L - [z]_0^L \right] =
\end{aligned}$$



TRATTO AB $0 \leq z \leq L$

$$M^{(r)}(z) = q \frac{z^2}{2} - \frac{2}{3} q L^2$$

$$M_A = -\frac{2}{3} q L^2$$

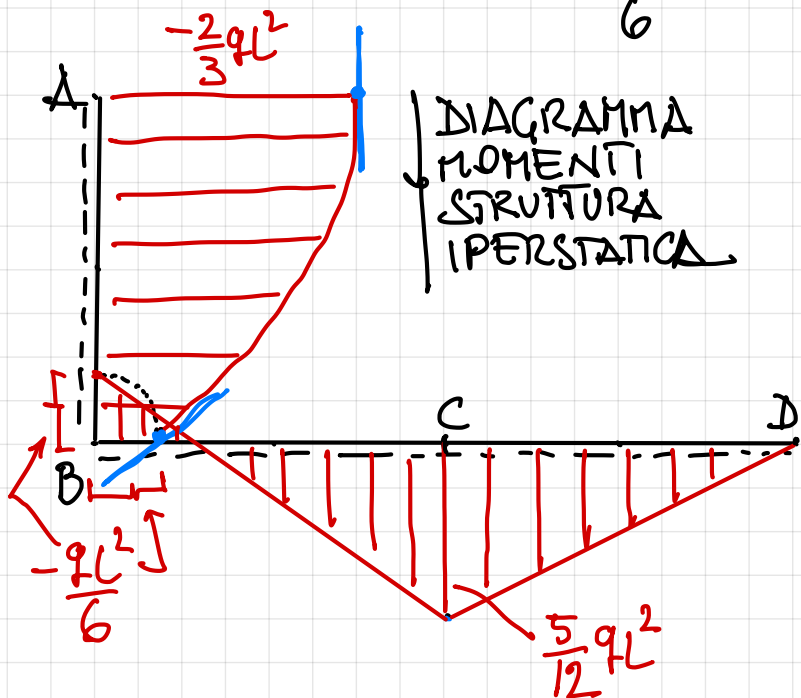
$$M_B = -\frac{q L^2}{6}$$

TRATTO BC $0 \leq z \leq L$

$$M^{(r)}(z) = -\frac{q L^2}{6} + \frac{7}{12} q L \cdot z$$

$$M_B = -\frac{q L^2}{6}$$

$$M_C = \frac{5}{12} q L^2$$



TRATTO CD $L \leq z \leq 2L$

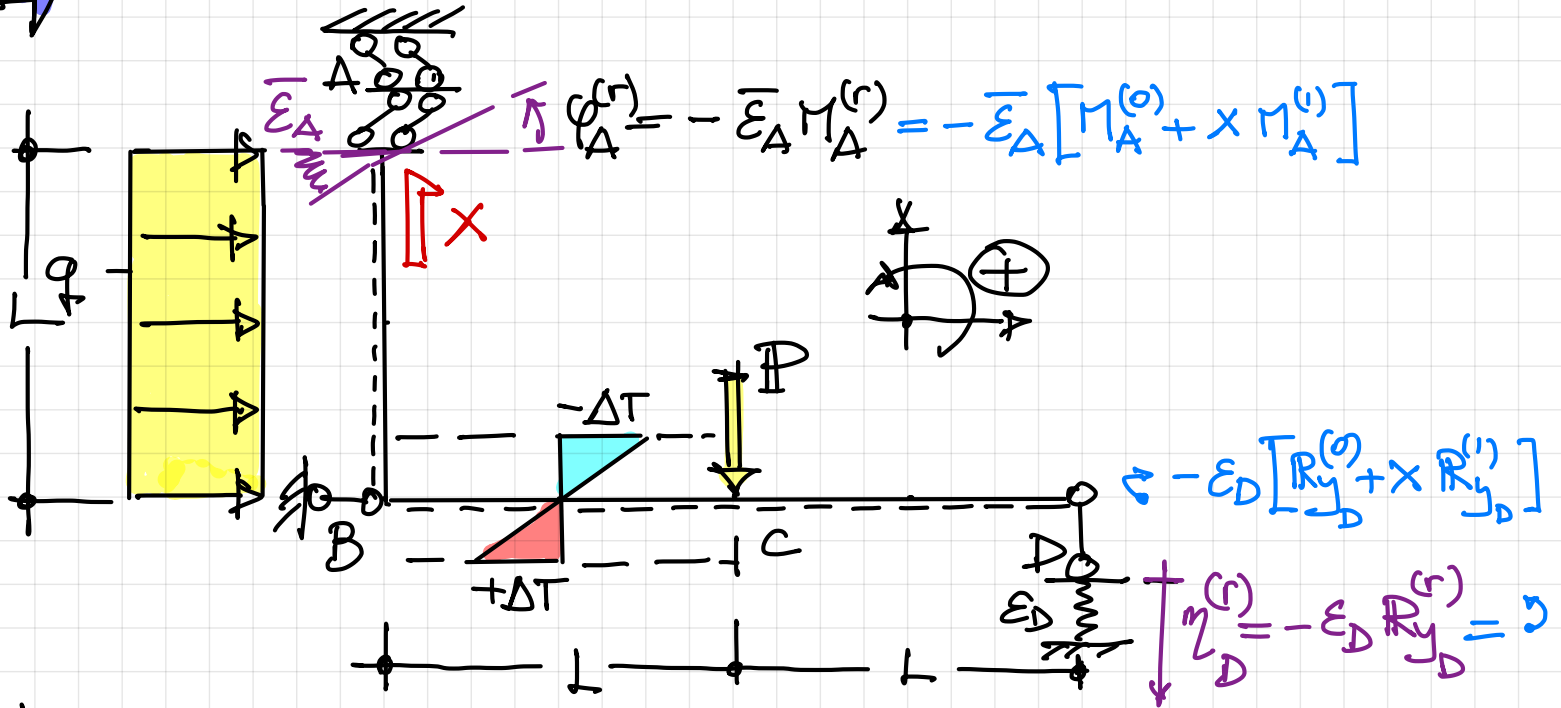
$$M^{(r)}(z) = \frac{5}{12} q L (2L - z)$$

$$M_C = \frac{5}{12} q L^2$$

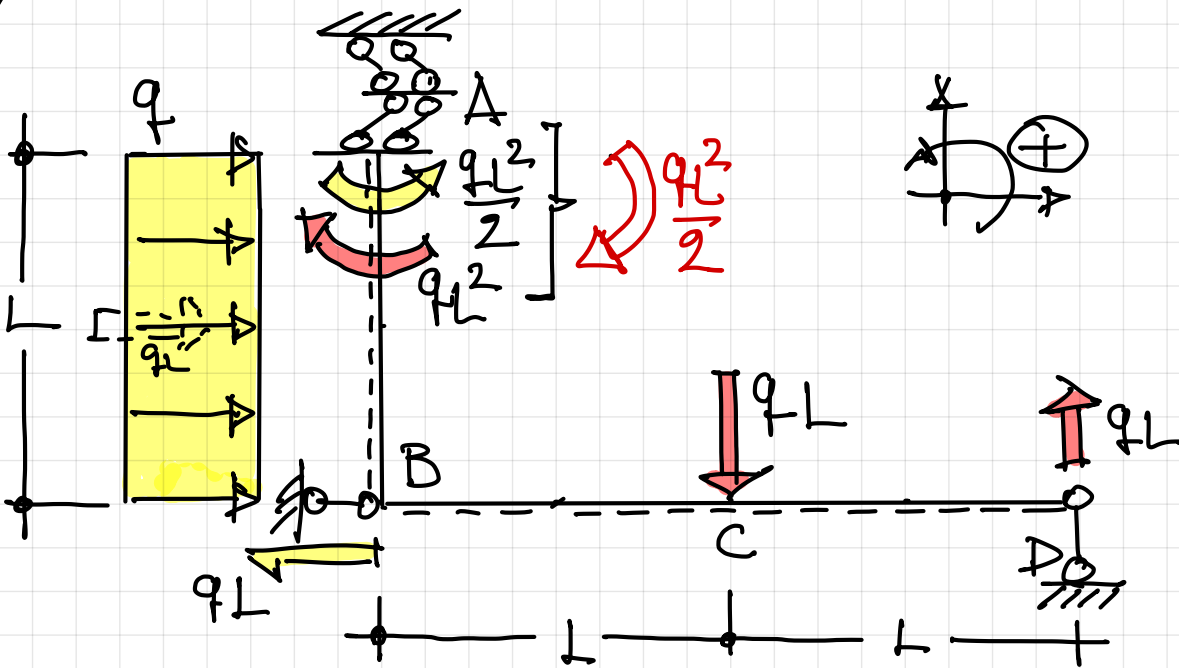
$$M_D = 0$$

SOLUZIONE #2

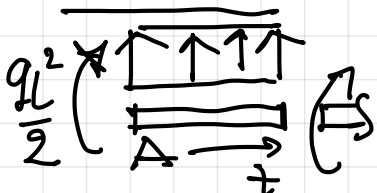
SISTEMA PRINCIPALE ISOSTATICO



Schema [0] SOLO CARICHI ESTERNI



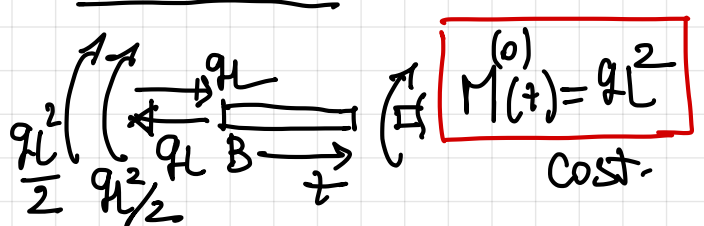
TRATTO AB $0 \leq z \leq L$



$$M^{(0)}(z) = \frac{qL^2}{2} + \frac{qz^2}{2}$$

$$M_A = \frac{qL^2}{2} \quad M_B = qL^2$$

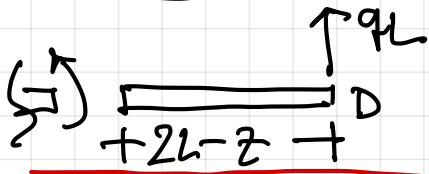
TRATTO BC $0 \leq z \leq L$



$$M^{(0)}(z) = qL^2$$

Cost.

TRATTO CD $L \leq z \leq 2L$



$$M^{(0)}(z) = qL(2L-z)$$

$$M_C = qL^2; M_D = 0$$

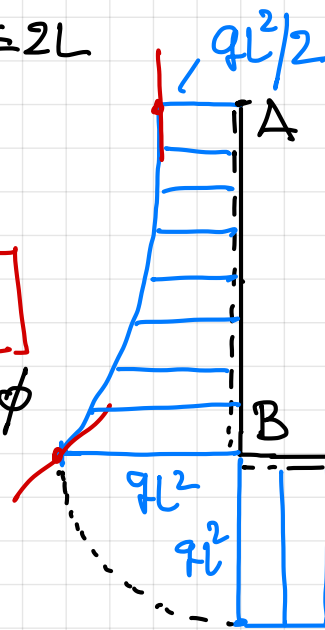
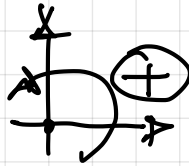
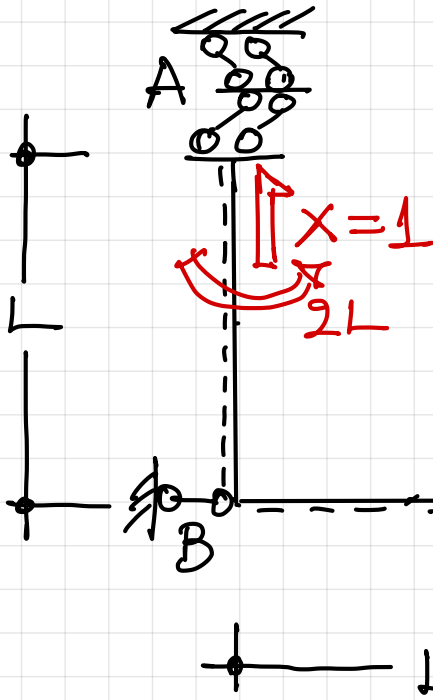


DIAGRAMMA $M^{(0)}(z)$

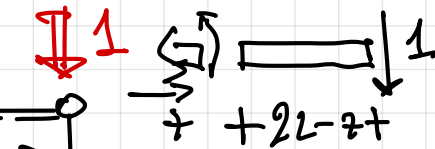
SCHEMA [1] SOLO $x=1$



TRATTO AB $0 \leq z \leq L$

$$M^{(1)}(z) = -2L \quad \text{cost}$$

TRATTO BD $0 \leq z \leq 2L$



$$M^{(1)}(z) = -(2L-z)$$

$$M_B = -2L; M_D = 0$$

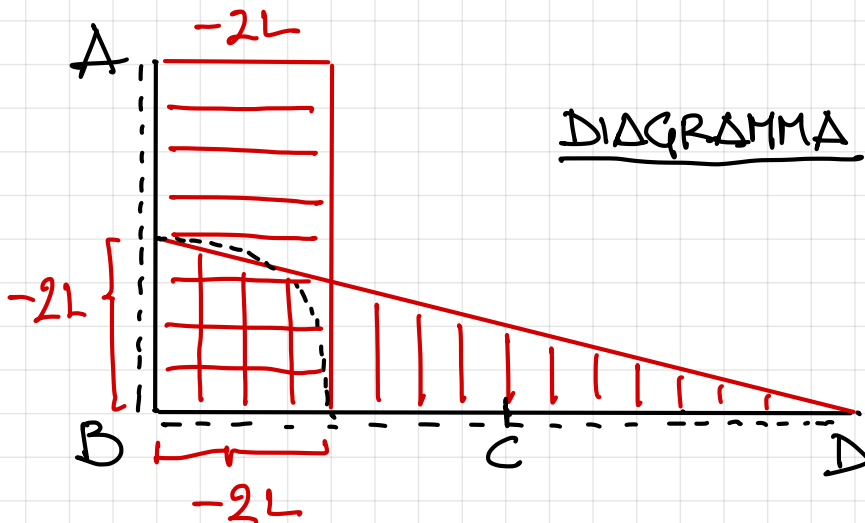


DIAGRAMMA $M^{(1)}(z)$

$$\Rightarrow \underline{Lve} = \sum_i X_i \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$= 1 \cdot \phi + M_A^{(1)} \varphi_A^{(r)} + R_{y_D}^{(1)} \eta_D^{(r)} =$$

$$= \underbrace{M_A^{(1)}}_{2L} (-\bar{\varepsilon}_A) \left[\underbrace{M_A^{(0)}}_{-\frac{qL^2}{2}} + x \underbrace{M_A^{(1)}}_{2L} \right] + \underbrace{R_{y_D}^{(1)}}_{-1} (-\varepsilon_D) \left[\underbrace{R_{y_D}^{(0)}}_{qL} + x \underbrace{R_{y_D}^{(1)}}_{-1} \right] =$$

$$= -\bar{\varepsilon}_A 2L \left[-\frac{qL^2}{2} + 2Lx \right] + \varepsilon_D [qL - x]$$

$$\Rightarrow \underline{Lvi} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI} dstr + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} dstr =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} dstr + \frac{x}{EI} \int_{str} [M^{(1)}]^2 dstr + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} dstr =$$

$$= \frac{1}{EI} \left\{ \int_0^L (-2L) \left[\frac{qL^2}{2} + \frac{qz^2}{2} \right] dz + \int_0^L -(2L-z) qL^2 dz + \right.$$

$$\left. + \int_L^{2L} -(2L-z) qL dz \right\} +$$

$$+ \frac{x}{EI} \left\{ \int_0^L 4L^2 dz + \int_0^{2L} (2L-z)^2 dz \right\} - \frac{\alpha \Delta T}{h} \int_0^L (2L-z) dz =$$

$$= \frac{1}{EI} \left\{ -qL^3 \left[z \right]_0^L - qL \left[\frac{z^3}{3} \right]_0^L - 2qL^3 \left[z \right]_0^L + qL^2 \left[\frac{z^2}{2} \right]_0^L \right.$$

$$\left. - 4qL^3 \left[z \right]_L^{2L} - qL \left[\frac{z^3}{3} \right]_L^{2L} + 4qL^2 \left[\frac{z^2}{2} \right]_L^{2L} \right\} +$$

$$+ \frac{x}{EI} \left\{ 4L^2 \left[z \right]_0^L + 4L^2 \left[z \right]_0^{2L} + \left[\frac{z^3}{3} \right]_0^{2L} - 4L \left[\frac{z^2}{2} \right]_0^{2L} \right\} +$$

$$- \frac{\alpha \Delta T}{h} \left\{ 2L \left[z \right]_0^L - \left[\frac{z^2}{2} \right]_0^L \right\} =$$

$$\begin{aligned}
 &= \frac{qL^4}{EI} \left[-1 - \frac{1}{3} - 2 + \frac{1}{2} - 4 - \frac{7}{3} + 6 \right] + \\
 &\quad + X \frac{L^3}{EI} \left[4 + \cancel{8} + \frac{8}{3} - \cancel{8} \right] - \frac{\alpha \Delta T}{h} \left[2L^2 - \frac{L^2}{2} \right] = \\
 &= -\frac{qL^4}{EI} \frac{19}{6} + X \frac{L^3}{EI} \frac{20}{3} - \frac{\alpha \Delta T}{h} \frac{3}{2} L^2 \quad \frac{-6-2-12(-3)-24}{6} \\
 &\quad \quad \quad -14 + (36) \\
 &\quad \quad \quad +39 - 58
 \end{aligned}$$



$L_{re} = L_{ri}$ fornisce:

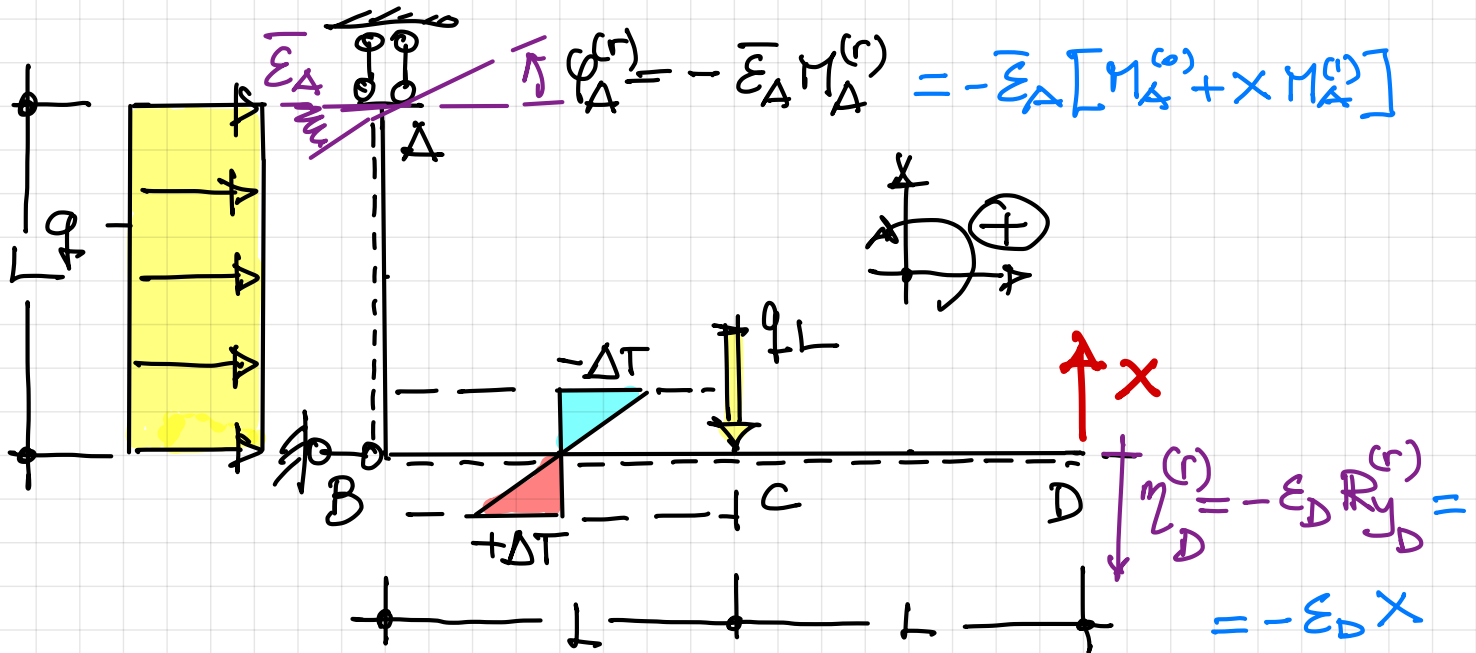
$$\begin{aligned}
 &-\bar{E}_A 2L \left[-\frac{qL^2}{2} + 2LX \right] + E_D \left[qL - X \right] = \\
 &= -\frac{qL^4}{EI} \frac{19}{6} + X \frac{L^3}{EI} \frac{20}{3} - \frac{\alpha \Delta T}{h} \frac{3}{2} L^2 \\
 &X \left[\frac{L^3}{EI} \frac{20}{3} + \underbrace{\bar{E}_A 4L^2}_{\frac{L}{EI}} + \underbrace{E_D}_{\frac{4L^3}{3EI}} \right] = + \frac{qL^4}{EI} \frac{19}{6} + \underbrace{\frac{\alpha \Delta T}{h} \frac{3}{2} L^2}_{\frac{qL^2}{EI}} + \underbrace{E_D qL}_{\frac{4L^3}{3EI}} + \underbrace{\bar{E}_A qL^3}_{\frac{L}{EI}} \\
 &\cancel{X} \frac{L^3}{EI} \left[\frac{20}{3} + 4 + \frac{4}{3} \right] = \frac{qL^4}{EI} \left[\frac{19}{6} + \frac{3}{2} + \frac{4}{3} + 1 \right] \\
 &\quad \quad \quad 12 \quad \quad \quad 7
 \end{aligned}$$

da cui $X = \frac{7}{12} qL$ OK! cfr. RV di pag. 6

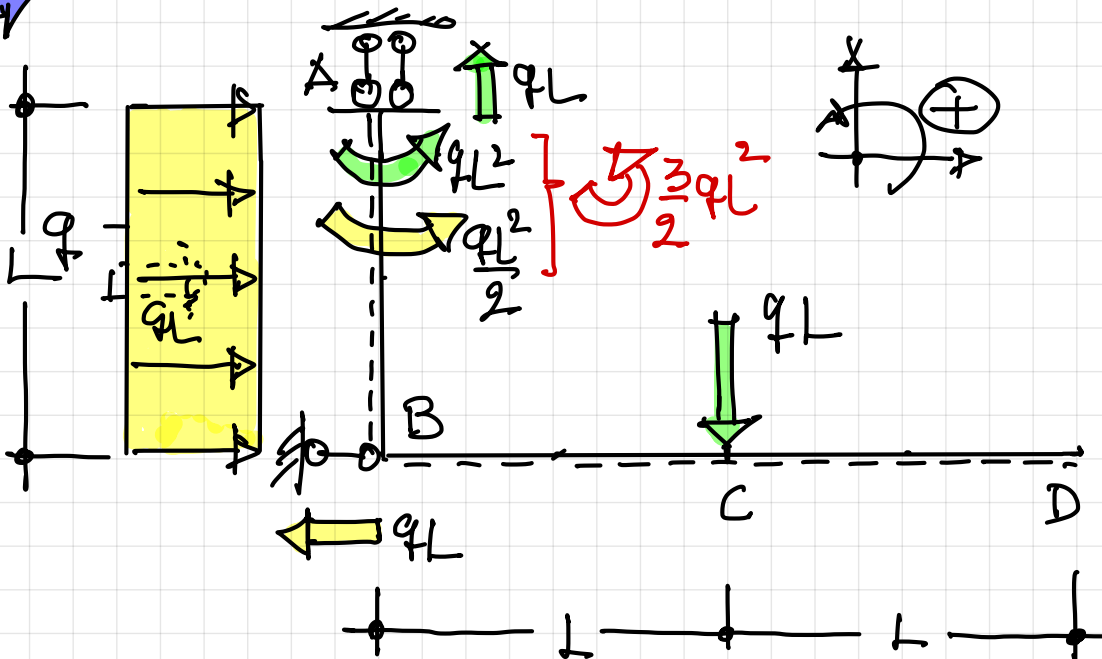
SOLUZIONE #3

11

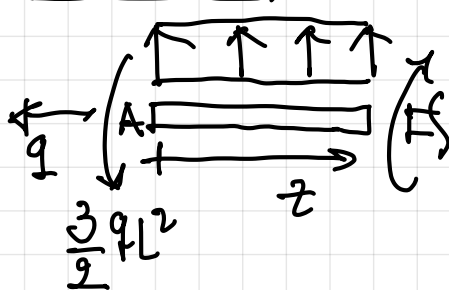
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO AB $0 \leq z \leq L$

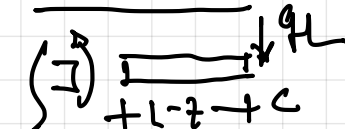


$$M^{(0)}(z) = \frac{qz^2}{2} - \frac{3}{2}qL^2$$

$$M_A = -\frac{3}{2}qL^2$$

$$M_B = -qL^2$$

TRATTO BC $0 \leq z \leq L$



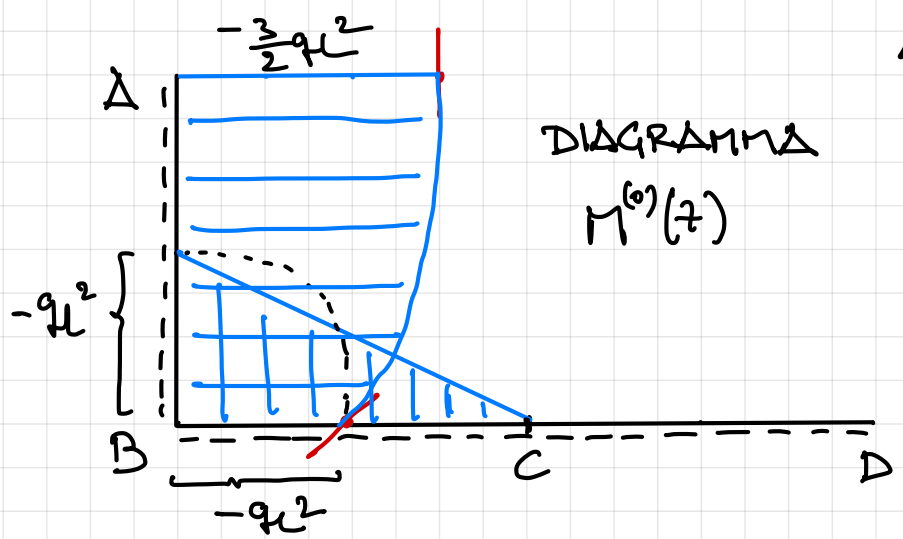
$$M^{(0)}(z) = -qL(L-z)$$

$$M_B = -qL^2$$

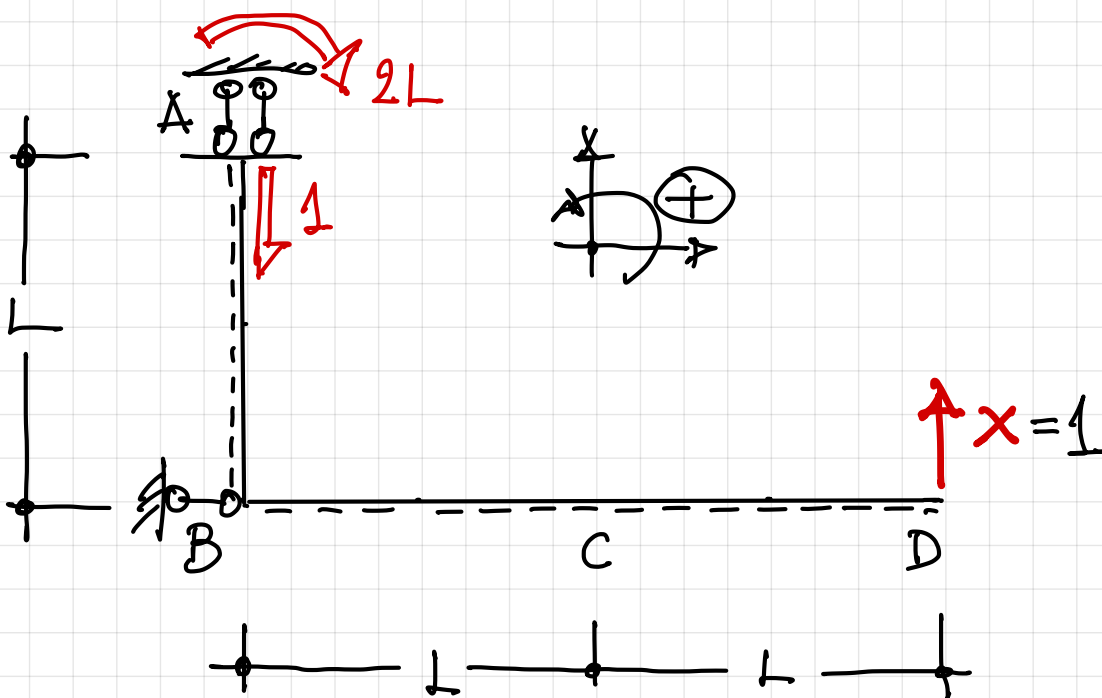
$$M_C = 0$$

TRATTO CD $L \leq z \leq 2L$

SCARICO $M^{(0)}(z) = \phi$



➡ SCHEMA [1] SOLO $x=1$

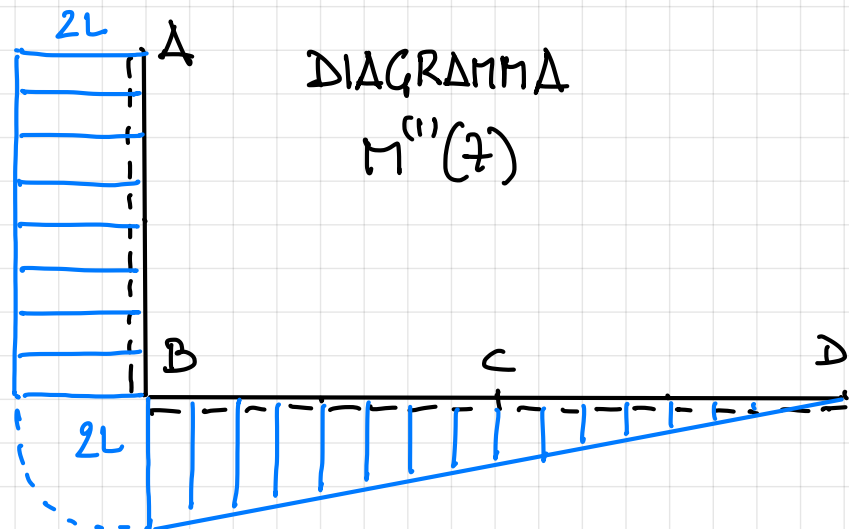


TRATTO AB $0 \leq x \leq L$

$M^{(1)}(x) = 2L \cos t$

TRATTO BD $0 \leq z \leq 2L$

$M^{(1)}(z) = 2L - z$
 $M_B = 2L$
 $M_D = \phi$



$$\Rightarrow \underline{L_{ve}} = \underbrace{X}_1 \underbrace{\eta_i^{(r)}}_1 + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon_D X} + \underbrace{M_A^{(1)}}_{-2L} (-\bar{\varepsilon}_A) \left[\underbrace{M_A^{(0)}}_{\frac{3}{2} q L^2} + X \underbrace{M_A^{(1)}}_{-2L} \right] =$$

$$= -\varepsilon_D X + 2L \bar{\varepsilon}_A \left[\frac{3}{2} q L^2 - 2L X \right]$$

$$\Rightarrow \underline{L_{vi}} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI} dsr + \int_{str} M^{(1)} \frac{\alpha \bar{\Delta T}}{h} dsr =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} dsr + \frac{X}{EI} \int_{str} [M^{(1)}]^2 dsr + \frac{\alpha \bar{\Delta T}}{h} \int_{str} M^{(1)} dsr =$$

$$= \frac{1}{EI} \left\{ \int_0^L 2L \left(\frac{qz^2}{2} - \frac{3}{2} q L^2 \right) dz + \int_0^L \underbrace{(2L-z)[-qL(L-z)]}_{-qL(2L^2 - 2Lz - zL + z^2)} dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L 4L^2 dz + \int_0^{2L} \underbrace{(2L-z)^2}_{4L^2 + z^2 - 4Lz} dz \right\} + \frac{\alpha \bar{\Delta T}}{h} \int_0^L (2L-z) dz =$$

$$= \frac{1}{EI} \left\{ qL \left[\frac{z^3}{3} \right]_0^L - 3qL^3 [z]_0^L - 2qL^3 [z]_0^L + 2qL^2 \left[\frac{z^2}{2} \right]_0^L + \right.$$

$$\left. + qL^2 \left[\frac{z^2}{2} \right]_0^L - qL \left[\frac{z^3}{3} \right]_0^L \right\} +$$

$$+ \frac{X}{EI} \left\{ 4L^2 [z]_0^L + 4L^2 [z]_0^{2L} + \left[\frac{z^3}{3} \right]_0^{2L} - 4L \left[\frac{z^2}{2} \right]_0^{2L} \right\} +$$

$$+ \frac{\alpha \bar{\Delta T}}{h} \left\{ 2L [z]_0^L - \left[\frac{z^2}{2} \right]_0^L \right\} =$$

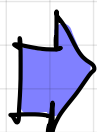
$$\begin{aligned}
 &= \frac{qL^4}{EI} \left[\frac{1}{3} - 3 - 2 + 1 + \frac{1}{2} - \frac{1}{3} \right] + \\
 &\quad + \frac{XL^3}{EI} \left[4 + \cancel{8} + \frac{8}{3} - \cancel{8} \right] + \frac{\alpha \Delta T}{h} \left[2L^2 - \frac{L^2}{2} \right] = \\
 &= \underbrace{-\frac{21}{6} \frac{qL^4}{EI} + \frac{20}{3} \frac{XL^3}{EI} + \frac{\alpha \Delta T}{h} \frac{3L^2}{2}}
 \end{aligned}$$

→ $L_{ve} = L_{vi}$ fornisce:

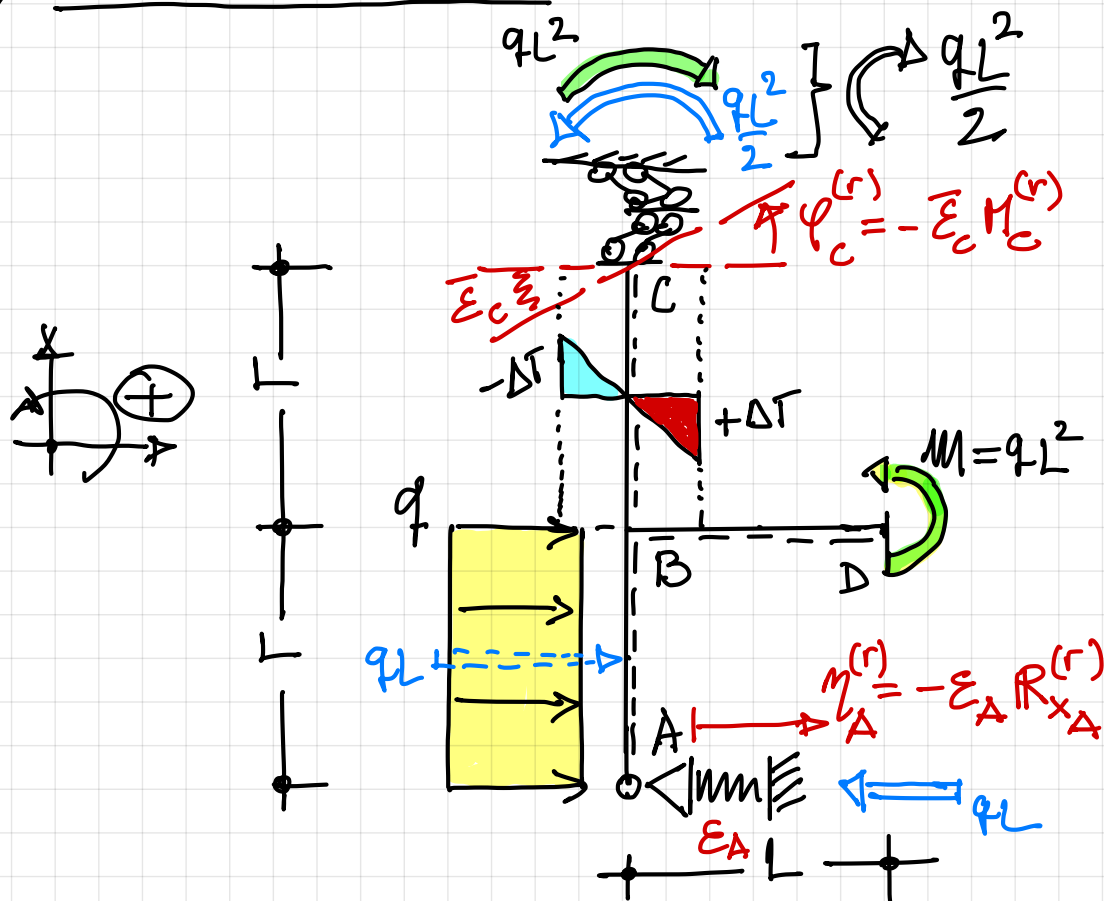
$$\underbrace{-\varepsilon_D X}_{\frac{4L^3}{3EI}} + 2L \underbrace{\varepsilon_A}_{\frac{L}{EI}} \left[\frac{3}{2} qL^2 - 2LX \right] = \underbrace{-\frac{21}{6} \frac{qL^4}{EI}}_{\frac{qL^2}{EI}} + \frac{20}{3} \frac{XL^3}{EI} + \underbrace{\frac{\alpha \Delta T}{h} \frac{3L^2}{2}}_{\frac{qL^2}{EI}}$$

$$X \cancel{\frac{L^3}{EI}} \left[\underbrace{\frac{20}{3} + \frac{4}{3} + 4}_{12} \right] = \cancel{\frac{qL^4}{EI}} \left[\underbrace{3 + \frac{21}{6} - \frac{3}{2}}_5 \right]$$

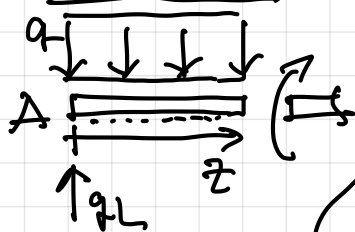
da cui $X = \frac{5}{12} qL$ OK! cfr. RV di pag. 6



STRUTTURA REALE



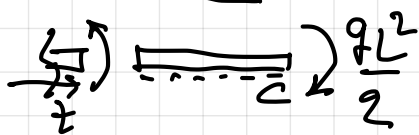
TRATTO AB $0 \leq z \leq L$



$$M^{(r)}(z) = qLz - \frac{qz^2}{2}$$

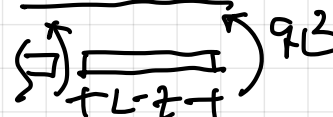
$$\begin{cases} M_A = 0 \\ M_B = \frac{qL^2}{2} \end{cases}$$

TRATTO BC $L \leq z \leq 2L$



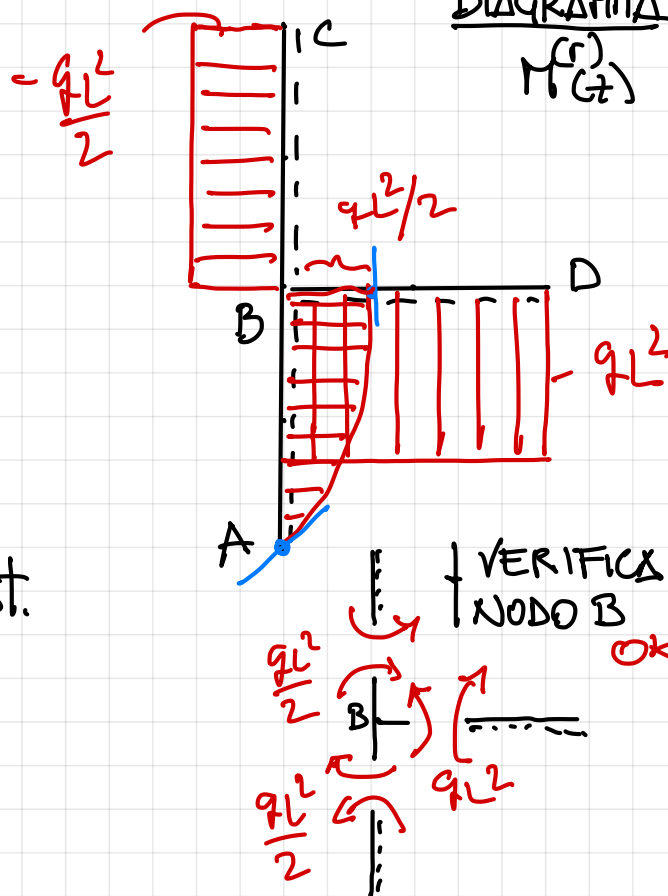
$$M^{(r)}(z) = -\frac{qL^2}{2} \text{ cost.}$$

TRATTO BD $0 \leq z \leq L$

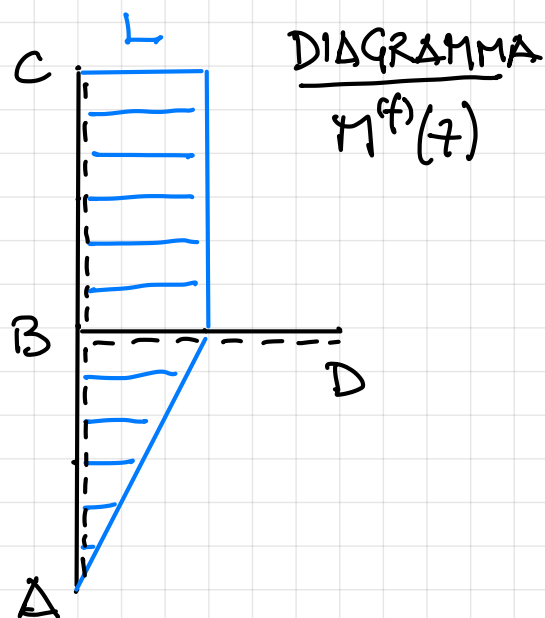
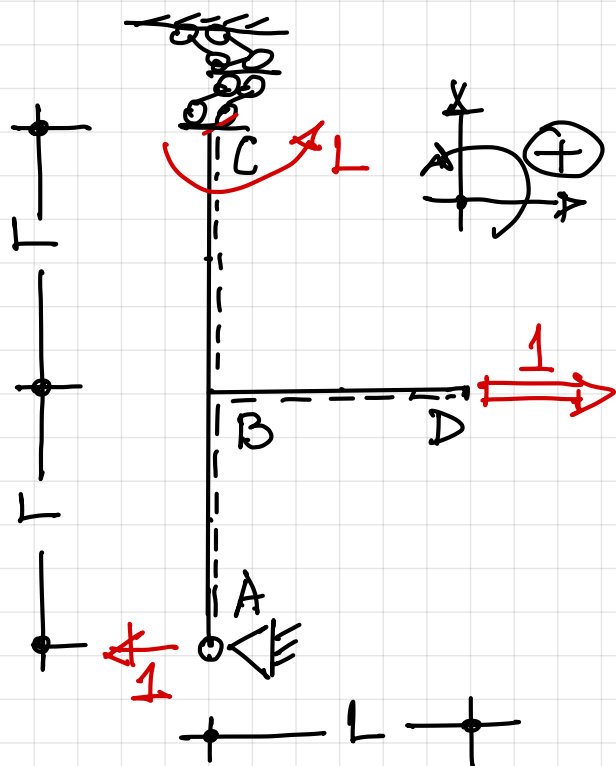


$$M^{(r)}(z) = qL^2 \text{ cost.}$$

DIAGRAMMA



STRUTTURA FITTIZIA PER IL CALCOLO DELLO SPOSTAMENTO ORIZZONTALE DELLA SEZIONE D



TRATTO AB $0 \leq z \leq L$

↑ 1

A \xrightarrow{z}

$M^{(f)}(z) = z$

$M_A = 0$
 $M_B = L$

TRATTO BC $L \leq z \leq 2L$

$\sum \rightarrow$

$M^{(f)}(z) = L$ cost.

TRATTO BD
SCARICO

$M^{(f)}(z) = 0$

➡ $L_{ve} = 1 \cdot \eta_D + \sum_j R_j^{(f)} \eta_j^{(r)} =$

$= \eta_D + R_{x_A}^{(f)} \eta_A^{(r)} + \overline{M}_C^{(f)} \varphi_C^{(r)} =$

$= \eta_D - \varepsilon_A qL + \overline{\varepsilon}_C \frac{qL^3}{2}$

$$\Rightarrow \underline{L_{vi}} = \int_{str} M^{(f)} \frac{M^{(r)}}{EI} dstr + \int_{str} M^{(f)} \frac{\alpha \Delta T}{h} dstr =$$

$$= \frac{1}{EI} \left\{ \int_0^L z \left[qLz - \frac{qz^2}{2} \right] dz + \int_L^{2L} L \left[-\frac{qL^2}{2} \right] dz \right\} + \frac{\alpha \Delta T}{h} \int_L^{2L} L dz =$$

$$= \frac{1}{EI} \left\{ qL \left[\frac{z^3}{3} \right]_0^L - \frac{q}{2} \left[\frac{z^4}{4} \right]_0^L - \frac{qL^3}{2} [z]_L^{2L} \right\} + \frac{\alpha \Delta T}{h} L [z]_L^{2L} =$$

$$= \frac{1}{EI} \left\{ \frac{qL^4}{3} - \frac{qL^4}{8} - \frac{qL^4}{2} \right\} + \frac{\alpha \Delta T}{h} L^2 =$$

$$= \underline{-\frac{qL^4}{EI} \frac{7}{24} + \frac{\alpha \Delta T \cdot L^2}{h}}$$

$$\Rightarrow L_{ve} = L_{vi} \text{ fornisce}$$

$$\eta_D - \underbrace{\varepsilon_A qL}_{\frac{2}{3} \frac{qL^3}{EI}} + \underbrace{\varepsilon_C \frac{qL^3}{2}}_{\frac{L}{EI}} = -\frac{qL^4}{EI} \frac{7}{24} + \underbrace{\frac{\alpha \Delta T \cdot L^2}{h}}_{\frac{13}{8} \frac{qL^2}{EI}}$$

da cui:

$$\eta_D = \frac{qL^4}{EI} \left[\frac{2}{3} - \frac{1}{2} - \frac{7}{24} + \frac{13}{8} \right] \Rightarrow \underline{X = \frac{3}{2} \frac{qL^4}{EI}} \left\{ \begin{array}{l} \text{POSITIVO!} \\ \text{VERSO} \\ \text{DESTRA} \end{array} \right.$$

$\frac{36}{24} = \frac{3}{2}$